CONTRIBUTION OF THE VARIOUS THERMODYNAMICAL PARAMETERS IN THE STUDY OF BLACK HOLE THERMODYNAMICS

A Thesis
Submitted for the Degree of
DOCTOR OF PHILOSOPHY
In Physics
In the faculty of science
B. N. Mandal University
Madhepura
2012

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ISBN: 978-93-7542-208-2

First Edition: April 2026

Foreword

It gives me pleasure to write the foreword of the book "Contribution Of The Various Thermodynamical Parameters In The Study Of Black Hole Thermodynamics" authored by Dr. Ranjan Prasad. The formtion of black holes and their properties form a field of study of highest theoretical interest. In recent years our knowledge of this subject has vastly increased and the field of research covers a large range of mass withour any limit. Recent astrophysical researches which are of nature of unsupported speculations indicate the existence of immensely large mass. The author of this research book has experience of the study of black holes and their themodynamical properties. Most of the properties of black holes have been theretocially explain based on the principle of thermodynamics. The readers of this book will be very much benefited in the basic knowledege of black holes.

Dr. S. N. Singh

Dedicated to my parents.

Certificate

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Date …………….

Sri Ranjan Prashad, Research Scholar, B. N. Mandal University, Madhepura has done his research work under my supervision and guidance for the Ph. D. degree. I certify that this thesis is a record of work done by him and that the contents of this thesis did not form a basic of award of any previous degree to the candidate or to the best of my knowledge to anybody else.

(Dr. Uma Shankar Choudhary)

Supervisor

Abstract

Thermodynamics of various black holes is studied by means of thermodynamic Riemannian curvatures. The thermodynamic Riemannian curvature is a scalar curvature of the Ruppeiner metric, which is defined as a metric tensor on the thermodynamic parameter space, mathematically it is the matrix of second derivatives (Hessian matrix) of the entropy with respect to the energy (mass of the black hole) and other extensive thermodynamic parameters such as the black hole's spin and the electric charge. In ordinary thermodynamics the Ruppeiner metric is a flat metric if the underlying statistical mechanical system is non interacting such as that of the ideal gas. Furthermore, the curvature scalar of the Ruppeiner metric diverges at the critical point. In this Thesis, various black hole families are investigated, i.e. the BTZ, the Reissner-Nordstrom (RN), the Reissner-Nordstrom-anti-desitter (Rnads), the Kerr and the kree-Newman (KN) black holes respectively and it is found that the BTZ and the RN black holes both have vanishing curvatures. The RNads black hole has a curved geometry and gives interpretable results-namely that its curvature diverges in the external limit and its matric changes signature where the themodynamical stability properties of the black hole change. The kerr black hole has a non-zero curvature scalar, which diverges at the external limit, whereas the Ruppeiner metric for KN black hole has non-trivial curvature and does not give raise any further conclusive remarks.

Acknowledgement

I Would like to place on record my sincere thanks to the following people who have directly or indirectly assisted me during the course of my entire Thesis Work.

To Dr. Uma Shankar Choudhary Associate Professor & Head, Dept of Physics, Nirmali College, Nirmali, Supaul for his Constant Supervision suggestion and encouragement during my research work.

To Dr. J. P. Choudhary, Head, University Dept. of Physics, B. N. Mandal University, Madhepura for his guidance during my research work.

To Dr. J. Thakur, Dean, Faculty of Science, B.N.M.U. Madhepura for his regular suggestion.

To Dr. Ashok Kumar, Associate Professor, University Dept. of Physics, B.N. Mandal University, Madhepura for his Suggestion and encouragement during my research work.

To Dr. Rajiv Sinha, Associate Professor & Principal M.L.T. College, Saharsha B.N. Mandal University, Madhepura for encouragement and suggestions from time to time during my research work.

To Dr. Vimal Sager, & Dr. D. P. Singh, University Dept. of Physics B.N.M.U. Madhepura for his suggestion during research work.

To Sri Santosh Kumar & Sri Maheswari Pd. Yadav University Dept. of Physics B. N. M. U. Madhepura for his official Support in my research work.

To my parents, and children who have sacrificed and support financially and morally. I feel that without their sacrifices and help this work could not have been completed.

Ranjan Prasad

B. N. MANDAL UNIVERSITY, LALOONAGAR
MADHEPURA.

Form

Guideline formulation of a Research proposal (SYNOPSIS) in the faculty of Science (Physics)

1. TITLE OF RESEARCH WORK/PROBLEM:-

"CONTRIBUTION OF THE VERIOUS THERMODYNAMICAL PARAMETERS IN THE STUDY OF BLACK HOLE THERMODYNAMICS"

2. STATEMENT OF THE RESEARCH PROBLEM:-

Thermodynamics of various black holes is studied by means of thermodynamic Riemannian curvatures. The thermodynamic of Riemannian curvature is a scalar curvature of the Ruppeiner metric, which is defined as a metric tensor on the thermodynamic parameter space, mathematically it is the matrix of second derivatives (Hessian matrix) of the entropy with respect to the energy (mass of the black hole) and other extensive thermodynamic parameters such as the black hole's spin and the electric charge. In ordinary thermodynamics the Ruppeiner metric is a flat metric if the underlying statistical mechanical system is non-interacting such as that of the ideal gas. Furthermore, the curvature scalar of the Ruppeiner metric diverges at the critical point. In this thesis, various black hole families are investigated, i.e., the BTZ, the Reissner-Nordstrom (RN), the Reissner-Nordstrom- anti-de Sitter (RNadS), the Kerr and the Kerr-Newman (KN) black holes respectively.

3. THEME AND AREA OF THE STUDY:-

The theme of this research works is to present a wide spread analysis of thermodynamic study of black hole. This work is also capable of computing different parameters of black hole.

4. REVIEW OF LITERATURE:

A brief overview of the studies already done in the area of the proposed study.

Many researchers and authors have made area of their significant contributions in them are this field. The prominent among at et. 25,1596 Christodoulou, D., Phys. Rev. (1971)., Falcke, H. and Hehl, F. W., The galactic black hole, IoP (United Kingdom 2003). Hawking, S. W., Phys. Rev. Lett. 26, 1344 (1971), Hawking S. W. Nature (London) 248, 30 (1974), Hehl. F. W. et. al., Blackholes. Theory and Observation, Springer (Germany, 1998). Lynden-Bell, D. and Lynden-Bell, R. M., Mon. Not. R. astr. Soc. 181, 405-409 (1977). Nulton, J. D. and Salamon, P., Phys. Rev. A3I, 2520 (1985). Ruppeiner, G., Rev. Mod. Phys. 67, 605 (1995). Savani, J., Princeton Weekly, R. M., http//www.livingreviews.org/Articles/Volume4/2001-6wald/, cited on 19 February 2003,. J. M. Maldacena, “Eternal black holes in anti-de-Sitter,” JHEP 04 (2003), L. Fidkowski, V. Hubeny, M. Kleban, and S. Shenker, “The black hole singularity in Ads/CFT,” JHEP02, 014 (2004), A. Strominger, “Black hole Entropy from near-horizon microstates, “JHEP 02, 009, (1998).

5. METHODOLOGY:

a. **Objective of the study:**
b. **Hypothesis, if any:**
c. **Tools to be-used in the collection of data:**
d. **Coverage-universe of the study sampling procedure and size, units of observation /study:**

The proposed research work is completely theoretical. The observed data and results will be compared with previously available in the literature. In this research work different Books and Journals will be consulted at various library.

6. CONTRIBUTION OF THE STUDY:

Contribution which the proposed study of expected to make to theory and methodology as well as its practical importance and national relevance should be indicated specially. Black hole thermodynamics finds great significant due to wide application in the study of black hole, planet, satellite and space research work.

7. TENTATIVE CHAPTERISATIOIN OF THE PROPOSED STUDY:

The tentative chaptalization of the proposed study is as follows:
Chapter-I: Introduction and review.
Chapter-II: Black holes and General Relativity.
Chapter-III: Thermodynamic Fluctuation theory.
Chapter-IV: Results.
Chapter-V: Summary and Conclusion.

8. REFERENCES:

1. Aman, J., Bengtsson, I and Pidokrajt, N., Geometry of Black hole Thermodynamics, gr-qc/0304015 and General Relativity and Gravitation 35(10)1733, (2003).
2. Ban~ados, M. et.al., Phys. Rev. Lett. 69, 1849 (1992).
3. J. M. Maldacena, “Eternal black holes in anti-de-Sitter,” JHEP 04 (2003) 021, hep-th/0106112.
4. L. Fidkowski, V. Hubeny, M. Kleban, and S. Shenker, “The black hole singularity in

AdS/CFT," JHEP 02, 014 (2004), hep-th/0306170.

5. S. W. Hawking and D. N. Page, "Thermodynamics of black holes in anti-de Sitter space," Commun. Math. Phys. 87,577 (1983).
6. A. Strominger, "Black hole entropy from near-horizon microstates," JHEP 02, 009, (1998) hep-th/9712251.
7. Ban~ados, M. et.al., Phys. Rev. D48,1506 (1993).
8. Benenson W, et. al. Hand Book of Physics, Springer (New York, 2002).
9. Hawking, H. W., Phys Rev Lett 26, 1344(1971).
10. Hawking, H. W., Nature (London) 248, 30, (1974).
11. Ruppeiner, G., Rev. Mod. Phys 67, 605, (1995).
12. Savani, J., Princeton Weekly Bulletin, April, 14, 1997.
13. Wald, R. M. http:// www.livingreviews.org/Articles/Volume-4/2001/6 wald, cited on Feb., 2003
14. Will, C. M.,http://wugrav.wustl.edu/people/CLIFF/exptgravity.html, cited on 18 March, 2003.

Signature of the Supervisor   Signature of the Candidate

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Content

CHAPTER 1
Introduction

A black hole an object confined to the interior of its Schwarzschild surface is one of the stringent objects of nature. Its space time is so strongly curved that no light can come out of it no matter can be ejected from it and no measuring rod can survive being put in to it Any kind of object that falls into it loses its separate identity. It is remembered only through the mass the charge the angular momentum and the linear momentum it contributed to the hole. How the physics of a black hole looks depends essentially on the choice of observer.

Suppose that the observer fallows the collapsing matter through its collapse down into the black hole. Then he will see it crushed to indefinitely high density and he himself will be turn apart by indefinitely merging tidal forces. No restraining force has the power to hold him away from this catastrophe once one has crossed the critical Schwarzschild surface. The final collapse occurs a finite time after the passage of this surface but it is inevitable time and space are interchanged inside a black in an unusual manner, (This is related to the fact that the element g_{11} and g_{44} of the Schwarzschild metric change sign when r passes through the value r_o) The direction of rereading proper time for the observer is now the direction of decreasing values of the radial coordinate r. The observer has no more power to return to a larger value of r (at least stay where he is) than we have power to turn back (or at least stop the hand same the clock at life. Hawking discovery [1,2] That black holes emit radiation precisely as would a black body with a temperature h/ 8πGM represents an important landmark in astrophysics. (We use units with $c=K_B=1$ and confine ourselves to Schwarzschild black holes) Before this, the interior of black holes was believed to be perpetually hidden to external observers since nothing could come out of it. No investigation in to the nature of black hole could be meaningful without the flow of some information out of it. This made mysterious the successful conjecture of Bekenstein [3], regarding the entropy of black hole, and of Bardeen, carter and Bardeen [4], Postulating the laws of black hole mechanics later recognized as thermodynamics. As shown by Hawking [5], the law of thermodynamics for black holes

$$Tds = dM \quad \cdots\cdots\cdots\cdots \quad (1.1)$$

Implies entropy 4 $\pi GM^2/h$ for the black hole in agreement with Bekenstein conjecture. It should be pointed out that both the temperature and the entropy of a black hole are inherently quantum concept which becomes meaningless in the classical ($h\rightarrow 0$) limit. Classically, black holes are the singular and result of the gravitational collapse of a sufficiently massive could of gas [6], remarkably black holes can be described by one parameter only on the Schwarzschild case the mass M and in this sense represent perfect bodies.

The thermodynamics of black hole (entropy temperature) will be impossible to understand if they really are classical singularities. However, It is possible that the classical singularity is an approximation to a phase of very high density of the order of $\frac{M_{pl}}{L_{Pl}^3}$where $M_{pl}= \sqrt{h/G}$is the planck mass and $L_{pl}= \sqrt{h/G}$is the planck length. If the quantum mechanics of the system is normal (i. e. if the metric is lorentzian), It would be very difficult to understand as to how the temperature of the matter is precisely constrained to be that given by Hawking formula. (we are adopting there the simple point of view that the Hawking temperature is the actual, physical temperature of whatever is "inside the black hole"). This apart, there are other difficulties. The super dense phase of matter has a volume of the order o $\left(\frac{M}{M_{pl}}\right) L_{Pl}^3$. The entropy density is, then of order $\frac{M}{M_{pl}L_{pl}^3} \sim \frac{M}{TL_{Pl}^3}$ .This does not imply a positive specific heat at constant volume, disagrees with third law of thermodynamics and does not correspond to any known system.

Black hole is some of the most exotic entities encountered in physics of the present time. The nature of space-time with in a black hole is enough to make the science of black hole seem more like a science fiction. Even more astounding are the connections of black hole physics with thermodynamics. Classically we would expect the black hole to be a perfectly dead star namely it should have an absolute zero as a physical temperature. But it was not so since Hawking has found a star line discovery that the black hole radiates thermally [7,8] Whereas Bekenstein Suggested that there is an entropy associated with the black hole [9] i.e., the black hole entropy. However, that the black hole has an entropy first arose from the realization that its event horizon surface area exhibits remarkable tendency to increase when undergoing any transformation as noticed by Floyed and penrose [10] and later supported by Christodoulou [11]. Hawking [12], was the first to give a general proof that the surface area of the black hole cannot decrease in any process and additionally he showed that two black holes coalesce the area of the resulting black hole cannot be smaller than the sum of the initial areas. Easily speaking, it is clear that changes in black hole generally occur in the direction of increasing, area. This reminds us of the second law of ordinary thermodynamics which state that changes of a closed thermodynamics system take place in the direction of increasing entropy. Historically physicists where not convinced about the validity the black hole thermodynamics before Hawking radiation was discovered.

Despite the written down laws of black hole thermodynamics, we have never been able to do any real experiment to verify them, all we can do is the gedanken experiment. It is thus legitimate to say we kind of perform gedanken experiment on the black holes thermodynamically by names of thermodynamic fluctuation theory using the language of Riemannian geometry in this thesis work. By the veritable of the thermodynamic Riemannian curvature (i.e., the curvature scalar) of the Ruppeiner metric of the black hole under consideration we hopefully will obtain some, new information on the black hole. We investigate also the Weinhold metric for each black hole which is the metric conformably related to the Ruppeiner metric, and it turns that it gives us a helping hand when the Ruppiner metric is too complicated to deal with. Throughout this report, we will set $G = h = c = k_B = 1$ unless otherwise stated.

These units are sometime referred to as geometrized units.

1.1 A Brief History of Black Holes

Neither the layman nor the specialist, in general, have any knowledge of the historical circumstances underlying the genesis of the idea of the Black Hole. Essentially, almost all and sundry simply take for granted the unsubstantiated allegations of some ostentatious minority of the relativists. Unfortunately, that minority has been rather careless with the truth and is quite averse to having its claims corrected, notwithstanding the documentary evidence on the historical record. Furthermore, not a few of that vainglorious and disingenuous coterie, particularly amongst those of some notoriety attempt to dismiss the testimony of the literature with contempt, and even deliberate falsehoods, claiming that history is of no importance. The historical record clearly demonstrates that the Black Hole has been conjured up by combination of confusion, superstition and ineptitude, and is sustained by widespread suppression of facts, both physical and theoretical. The following essay provides a brief but accurate account of events. It has frequently been alleged by theoretical physicists [6,13] that Newton's theory of gravitation either predicts or adumbrates the black hole. This claim stems from a suggestion originally made by John Michell in 1784 that if a body is sufficiently massive, "all light emitted from such a body would be made to return to it by its own power of gravity". The great French scientist, P. S. de Laplace, made a similar conjecture in the eighteen century and undertook a mathematical analysis of the matter.

However, contrary to popular and frequent expert opinion, the Michell-Laplace dark body, as it is actually called, is not a black hole at all. The reason why is quite simple.

For a gravitating body we identify an escape velocity. This is a velocity that must be achieved by an object to enable it to leave the surface of the host body and travel out to infinity, where it comes to rest. Therefore, it will not fall back towards the host. It is said to have escaped the host. At velocities lower than the escape velocity, the object will leave the surface of the host, travel out to a finite distance where it momentarily comes to rest, then fall back to the host. Consequently, a suitably located observer will see the travelling object twice, once on its journey outward and once. Its return trajectory. If the initial velocity is greater than or equal to the escape velocity, an observer located outside the host, anywhere on the trajectory of the travelling object, will see the object just once, as it passes by on its outward unidirectional journey. It escapes the host. Now, if the escape velocity is the speed of light, this means that light can leave the host and travel out to infinity and come to rest there. It escapes the host. Therefore, all observers located anywhere on the trajectory will see the light once, as it passes by on its outward journey. However, if the escape velocity is greater than the speed of light, then light will travel out to a finite distance, momentarily come to rest, and fall back to the host, in which case a suitably located observer will see the light twice, once as it passes by going out and once upon its return. Furthermore, an observer located at a sufficiently large and finite distance from the host will not see the light, because it does not reach him. To such an observer the host is dark: a Michell-Laplace dark body. But this does not mean that the light cannot leave the surface of the host. It can, as testified by the closer observer. Now, in the case of the black hole, it is claimed by the relativists that no object and no light can even leave the event horizon (the "surface") of the black hole. Therefore, an observer, no matter how close to the event horizon, will see nothing. Contrast this with the escape velocity for the Michell-Laplace dark body where, if the escape velocity is the speed of light, all observers located on the trajectory will see the light as it passes out to infinity where it comes to rest, or when the escape velocity is greater than the speed of light, so that a suitably close observer will see the light twice, once when it goes out and once when it returns. This is completely opposite to the claims for the black hole. Thus, there is no such thing as an escape velocity for a black hole, and so the Michell-Laplace dark body is not a black hole. Those who claim the Michell-Laplace dark body a black hole have not properly understood the meaning of escape velocity and have consequently been misleading as to the nature of the alleged event horizon of a black hole. It should also be noted that nowhere in the argument for the Michell-Laplace dark body is there gravitational collapse to a point-mass, as is required for the black hole.

The next stage in the genesis of the black hole came with Einstein's General.

1.2 Theory of Relativity

Einstein himself never derived the black hole from his theory and never admitted the theoretical possibility of such an object, always maintaining instead that the proposed physical basis for its existence was incorrect. However, he was never able to demonstrate this mathematically because he did not understand the basic geometry of his gravitational field. Other theoreticians obtained the black hole from Einstein’s equations by way of arguments that Einstein always objected to. But Einstein was over-ruled by his less cautious colleagues, who also failed to understand the geometry.

1.3 Einstein’s gravitational field

The solution to Einstein’s field equations, from which the black hole has been extracted, is called the “Schwarzschild” solution, after the German astronomer Karl Schwarzschild, who, it is claimed by the experts, first obtained the solution and first predicted black holes, event horizons, and Schwarzschild radii, amongst other things. These credits are so common place that it comes as a surprise to learn that the famous “Schwarzschild” solution is not the one actually obtained by Karl Schwarzschild, even though all the supposed experts and all the text-books say so. Furthermore, Schwarzschild did not breathe a world about black holes because his true solution does not allow them.

Shortly after Einstein published the penultimate version of his theory of gravitation in November 1915, Kar Schwarzschild [14] obtained an exact solution for what is called the static vacuum field of the point-mass. At that time Schwarzschild was at the Russian Front, where he was serving in the German army, and suffering from a rare skin disease contracted there. On the 13th January 1916, he communicated his solution to Einstein, who was astonished by it. Einstein arranged for the rapid publication of Schwarzschild’s paper. Schwarzschild communicated a second paper to Einstein on the 24th February 1916 in which he obtained an extract solution for a sphere of homogeneous and incompressible fluid. Unfortunately, Schwarzschild succumbed to the skin disease, and died about May 1916, at the age of 42.

Working independently, Johannes Drose [15] obtained an exact solution for the vacuum field of the point-mass. He communicated his solution to the great Dutch scientist H. A. Lorentz, who presented the solution to the Dutch Royal Academy in Amsterdam at a meeting on the 27th May 1916. Droste’s paper was not published until 1917.By then Droste had learnt of Schwarzschild’s solution and therefore included in his paper a footnote in acknowledgement. Droste anticipated the mathematical procedure that would later lead to the black hole, and correctly pointed out that such a procedure is not permissible, because it would lead to a non-static solution to a static problem. Contra-hype?

Next came the famous “Schwarzschild” solution, actually obtained by the great German mathematician David Hilbert [16], in December 1916, a full year after Schwarzschild obtained his solution. It bears a little resemblance to Schwarzschild’s solution. Hilbert’s solution has the same form as Droste’s solution, but differs in the range of values allowed for the incorrectly assumed radius variable describing how far an object is located from the gravitating mass. It is this incorrect range on the incorrectly assumed radius variable by Hilbert that enabled the black hole to be obtained. The variable on the Hilbert metric, called a radius by the relativists, is in fact not a radius at all, being instead a real-valued parameter by which the true radii in the spacetime manifold of the gravitational field are rightly calculated. None of the relativists have understood this, including Einstein himself. Consequently, the relativists have never solved the problem of the gravitational field. It is amazing that such a simple error could produce such a gigantic mistake in its wake, but that is precisely could produce such a gigantic mistake in its wake, but that is precisely what the black hole is – a mistake for enormous proportions of course, the black hole violates the static nature of the problem, as pointed out by Droste, but the black hole theoreticians have ignored this important detail.

The celebrated German mathematician, Herman Wey1 [17], obtained an exact solution for the static vacuum field of the point-mass in 1917, by a very elegant method. He derived the same solution that Droste had obtained.

Immediately after Hilbert's solution was published there was discussion amongst the physicists as to the possibility of gravitational collapse into the nether world of the nascent black hole. During the Easter of 1922. the matter was considered at length at a meeting at the College de France, with Einstein in attendance.

In 1923 Marcel Brillouin [18] obtained an exact solution by a valid transformation of Schwarzschild's original solution. He demonstrated quite rigorously, in relation to his particular solution, that the mathematical process, which later spawned the black hole, actually violates the geometry associated with the equation describing the static gravitational field for the point-mass. He also demonstrated rigorously that the procedure leads to a non-static solution to a static problem, just as Droste had pointed out in 1916, contradicting the very statement of the initial problem to be solved what is the gravitational field associated with a spherically symmetric gravitating body, where the field is unchanging in time (static) and the space-time outside the body is free of matter (i. e. vacuum), other than for the presence of a test particle of negligible mass?

In mathematical terms, those solutions obtained by Schwarzschild, Droste and Weyl, and Brillouin, are mutually consistent, in that they can be obtained from one another by an admissible transformation of coordinates. However, Hilbert's solution is inconsistent with their solutions because it cannot be obtained from them or be converted to one of them by an admissible transformation of coordinates. This fact alone is enough to raise considerable suspicions about the validity of Hilbert's solution. Nonetheless, the relativists have not recognized this problem either, and have carelessly adopted Hilbert's solution, which they invariably call "Schwarzschild's" solution, which of course, it is certainly not.

In the years following, a number of investigators argued, in one way or another, that the "Schwarzschild" solution, as Hilbert's solution became known and Schwarzschild's real solution neglected and forgotten, leads to the bizarre object now called the black hole. A significant subsequent development in the idea came in 1949, when a detailed but erroneous mathematical study of the question by the Irish mathematical physicist J. L. Synge [19], was read before the Royal Irish Academy on the 25th April 1949, and published on the 20th March 1950. The study by Synge was quite exhaustive but being based upon false premises its conclusions are generally false too. Nonetheless, this paper was hailed as a significant breakthrough in the understanding of the structure of the space-time of the gravitational field.

It was in 1960 that the mathematical description of the black hole finally congealed, in the work of M. D. Kruskal [20] in the USA, and independently of G. Szekeres [21] in Australia. They allegedly found a way of mathematically extending the "Schwarzschild" solution into the region of the nascent black hole. The mathematical expression, which is supposed to permit this, is called the Kruskal-Szekeres extension. This formulation has become the cornerstone of modern relativists and is the fundamental argument upon which they rely for the theoretical justification of the black hole, which was actually christened during the 1960's by the American theoretical physicist, J. A. Wheeler, who coined the term. Since about 1970 there has been an explosion in the number of people publishing technical research papers, textbooks and popular science books and articles on various aspects of General Relativity. A large proportion of this includes elements of the theory of black holes. Quite a few are dedicated exclusively to the black hole. Not only is there now a simple black hole with a singularity, but also naked singularities, black holes without hair, super massive black holes at the centers of galaxies, black hole quasars, black hole binary systems, colliding black holes, black hole x-ray sources, charged black holes, rotating black holes, charged and rotating black holes, primordial black holes, mini black holes, evaporating black holes, wormholes, and other variants, and even white holes! Black holes are now "seen" everywhere by the astronomers, even though no one has ever found an event horizon anywhere. Consequently, public opinion has been persuaded that the black hole is a fact of Nature and that anyone who questions the contention must be a crackpot. It has become a rather lucrative business, this black hole. Quite a few have made fame and fortune peddling the shady story.

Yet it must not be forgotten that all the arguments for the black hole are theoretical, based solely upon the erroneous Hilbert solution and the meaningless Kruskal-Szekeres extension on it. One is therefore lead to wonder what it is that astronomers actually "see" when they claim that they have found yet another black hole here or there.

Besides the purely mathematical errors that mitigate the black hole, there are also considerable physical arguments against it, in addition to the fact that no event horizon has ever been detected.

What does a material point mean? What meaning can there possibly be in the notion of a material object without any spatial extension? The term material point (or point mass) is an oxymoron. Yet the black hole singularity is supposed to have mass and no extension. Moreover, there is not a single shred of experimental evidence to even remotely suggest that Nature makes material points. Even the electron has spatial extent, according to experiment, and to quantum theory. A "point" is an abstraction, not a physical object. In other words, a point is a purely mathematical object. Points and physical objects are mutually exclusive by definition. No one has ever observed a point, and no one ever will because it is unobservable, not being physical. Therefore, Nature does not make material points. Consequently, the theoretical singularity of the black hole cannot be a point-mass.

It takes an infinite amount of observer time for an object, or light, to reach the event horizon, irrespective of how far that observer is located from the horizon. Similarly, light leaving the surface of a body undergoing gravitational collapse, at the instant that it passes its event horizon, takes an infinite amount of observer time to reach an observer, however far that observer is from the event horizon. Therefore, the black hole is undetectable to the observer since he must wait an infinite amount of time to confirm the existence of an event horizon. Such an object has no physical meaning for the observer. Furthermore, according to the very same theoreticians, the Universe started with a Big Bang, and that theory gives an alleged age of 14 billion years for the Universe. This is hardly enough time for the black hole to form from the perspective of an external observer. Consequently, if black holes exist they must have been created at the instant of the Bang. They must be primordial black holes. But that is inconsistent with the Bang itself, because matter at that "time", according to the Big Bang theoreticians, could not form lumps. Even so, they cannot be detected by an external observer owing to the infinite time needed for confirmation of the event horizon. This now raises serious suspicions as to the validity of the Big Bang, which is just another outlandish theory, essentially based upon Friedmann's expanding Universe solution, not an established physical reality as the astronomers would have us believe, despite the now commonplace alleged observations they adduce to support it.

At first sight it appears that the idea of a binary system consisting of two black holes, or a hole and a star, and the claim that black holes can collide, are physical issues. However, this is not quite right, notwithstanding that the theoreticians take them as well-defined physical problems. Here are the reasons why these ideas are faulty. First, the black hole is allegedly predicted by General Relativity. What the theoreticians routinely fail to state clearly is that the black hole comes from a solution to Einstein's field equations when treating of the problem of the motion of a test particle of negligible mass in the vicinity of a single gravitating body. The gravitational field of the test particle is considered too small to affect the overall field and is therefore neglected. Therefore, Hilbert's solution is a solution for one gravitating body interacting with a test particle. It is not a solution for the interaction of two or more comparable masses. Indeed, there is no known solution to Einstein's field equations for more than one gravitating body. In fact, it is not even known if Einstein's field equations actually admit of solutions for multi-body configurations. Therefore, there can be no meaningful theoretical discussion of black hole binaries or colliding black holes, unless it can be shown that Einstein's field equations contain, hidden within them, solutions for such configurations of matter. Without at least an existence theorem for multi-body configurations, all talk of black hole binaries and black hole collisions is twaddle [22]. The theoreticians have never provided an existence theorem.

It has been recently proved that the black hole and the expanding Universe are not predicted by General Relativity at all [23, 24], in any circumstances. Since the Michel Laplace dark body is not a black hole either, there is no theoretical basis for it whatsoever.

Modern astrophysics is opening as a book of puzzles to the scientists. The black hole is one of these latest puzzles of astrophysics. Although the theory of general relativity and gravitation predicts the inevitable formation of black hole in the galaxy, they have not yet been observationally demonstrated. If, however, they do form according to the prediction of the theory, there may be as many as a billion or so of them in our galaxy as such, they should have a tremendous importance in the galactic phenomena, and fortunately, this aspect has not remained ignored by the theoretical physicists. What are then these black holes and how can they be formed? Why are they so called and what possibilities are there to actually "observe them? These questions and the like be shall discuss briefly in the present section.

We have already seen that stars with masses less then about 1.4 M_0 invariably end their life as white dwarfs. These starts meet their demise peacefully. But any star having a mass greater than 1.5 M_0 has a chance to explode violently at a certain stage of its evolution, unless the catastrophe is prevented by rotation of the star or loss of a part its mass before the catastrophic stage is reached. If the initial mass of the exploding star is not very high, say not greater than 10 M_0 then after the surface layers are blown off, the core will contract to from a stable neutron star in which the pressure of the degenerate neutrons counteracts the gravitational onslaught. The density of the stellar material at this stage of the order of $10^{14\text{-}15}$ gm cm^{-3}. If, however, the original star is more massive, then after the explosion even the degenerate neutrons core may not be able to resist further gravitational contraction. Even if such a star does not explode for some favorable reasons, it will undergo the process of construction. Because these massive stars quickly use up their nuclear fuel (their life-time is 10^7 years), they start cooling down. So they star will constant and during this course some of its potential energy will be transformed to heat. This fresh quota of heat, however, is not enough to prevent the collapse, and therefore, the star continues to collapse with gathering speed. We shall assume that we are dealing all along with a spherical star. As the collapse starts, if the star cannot shed of most of its mass, the collapse continues with increasing speed thereby producing more and more intense gravitation field and reducing the size of the star. In the process a stage is reached when the radius R_S of the star of mass M is given by

$$R_S=\frac{2GM}{c^2} \quad \text{..} \quad (1.2)$$

Where c is the speed of light. R_S is called the Schwarzschild radius. When this radius has been attained, the object will have such a strong gravitational field that it will prevent even light from escaping out of its influence. In Newtonian mechanics also one finds the same result that at the radius R_S the velocity of escape from the object equals the velocity of light. Since photons cannot escape, no event occurring within the Schwarzschild radius can be communicated outside this radius. The object is therefore lost of the outside universe expect that its strong gravitational field will be felt by other nearby objects. The Schwarzschild radius has therefore been called the event horizon (meaning that any event within it remains hidden from view as one below the horizon) and the resulting objects as the black hole.

Even this stage, not only that the collapse does not cease, but it continues with appalling speed. In a spherically collapsing star all the material will simultaneously reach the center within a period of about 1sec after crossing the event horizon. The entire mass thus shrinks to a point of infinite density, which forms a singularity. The singularity will be formed even if the collapse is not radically symmetric. An understanding of the nature and physics of this singularity has of late becomes a stumbling block to physicists and relativists alike.

Equation (1.2) shows that the radius of the event horizon is proportional to the mass of the collapsed object. The Schwarzschild radius for the sun is 2.67 km and the minimum density attained by the material at this stage is ~ 10^6 gm cm^{-3}, if we assume that the percentage of mass converted into energy is not significant. For a star of 10 M_0 these quantities are respectively ~ 27 km and ~10^{-14} gm cm^{-3}, while for a massing galactic nucleus of mass 10^9 M_0 they are respectively~ 10^{-4}pc and 10^{-2} gm cm^{-3}, the last value being only about 1 per cent of the average density of the Sun.

When the event horizon has formed, light from within cannot move out, while that from the outside neighborhood of the horizon will propagate, being strongly red shifted by the intense gravitation pull of the black hole. Not only that, everything both matter and light that may encounter the event horizon will be sucked in.

A clear understanding of the black hole physics and mechanics posed serious problems has been opened to meaningful theoretical investigations, thanks to the very significant works in the field by Penrose, Hawking, Bardeen, Lynden-Bell, Chandrasekhar and several others. It is now understood that a black hole may capture mass from the surrounding space by its intense gravitational pull. The probability for such capture becomes almost certain, if the black hole be the member of a binary system consisting of a normal star. Owing to the intense gravitational pull the black hole will them draw gas from the upper atmosphere of the companion. This gas will from a rotating disk around the black hole by virtue of the conservation of angular momentum which the captured matter originally possessed. Frictional force will dissipate the kinetic energy of matter, particularly of that in the inner part of the disk, whereby the centrifugal balance will be upset and the matter will spiral towards the black hole to be sucked in by it ultimately, in the same manner as atmospheric friction brings down the artificial satellites to the surface of the Earth. The falling matter, before being sucked in, attains very high temperature so as to radiate intensely in very high frequencies, particularly, in X-rays. Calculations show that about 6 per cent or more of the total rest-mass energy of the in falling matter may be converted to radiation energy during the process. This is about 8 times more efficient than the energy generation process by nuclear transmutation. Thus gravitational energy generation process is much more efficient than any other known process, except that of complete annihilation by matter-antimatter which is 100 per cent efficient.

The works of Penrose and Hawkins have shown that a still more efficient process of gravitational radiation can be achieved when two black holes comes in contact and merge together. In this case, the two event horizons also merge together to be enveloped by a common event horizon. When this happens, the theory predicts that the total surface area of the event horizon of the final black hole will increase. In an ideal case, let the two black holes be of equal mass, M, and the resulting black hole be mass M'. Now since the surface area of the event horizon is proportional to the square of the mass (the radius being proportional to the mass), we should have, according to the above theory,

$$M^{-2} > 2M^2$$

Whence,

$$M' > \sqrt{2M} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1.3)$$

Which corresponds to a maximum efficient of $\left(1 - \frac{1}{\sqrt{2}}\right) \approx 0.29$ for the conversion of original rest-mass energy to the radiation energy. This is upper limit of conversion efficiency in the case of no rotating black holes. It has been proved also that under certain conditions, if a mass is accreted by a rotating black hole, then before it is finally sucked in, about 43 per cent of its rest-mass energy may be converted to radiant energy; and this is the maximum limit of conversion efficiency so far known.

Such highly efficient mechanism for energy generation is called for, in order to explain the observed radiation from radio galaxies, quasars and intense X-ray sources. So black holes, although not yet observed, may come out as the final answer to the energy problem that has been seriously intriguing the physicists for more than a decade.

Lastly, we can ask-What prospect is there for a black hole to be observed and where should we look for it? If the way we have described for the formation of black holes is correct, there may be at least 10^9 black holes in the Galaxy. But the most likely place to detect them is in the massive binary system, from both the aspect of intense gravitation pull as well as high frequency radiation. We have already noticed that intense X-ray radiation result when mass is accreted by a black hole. So if in a binary system one of the components be a normal star while the other has become a black hole, the former will move in an orbit around the latter which will be optically invisible, but will emit copious flux of X-rays.

This condition is exactly fulfilled by the most intense X-ray source Cygnus X-1. It is now believed that Cygnus X-1 belongs to a binary system having its companion as a normal B0Iab super giant. Reliable calculations have indicated that the mass of the super-giant lies in the range (20-30) M_0 while that of the X-ray source lies in the range (3.4-8) M_0 Such a high mass of the X-ray source lends belief that it is not a neutrons star but a black hole. Such a belief is supported by the observation that the X-ray brightness of Cygnus X-1 varies over a period of 0.1 second. This indicates that the radius of the source is smaller than $3x10^4$ km, which is very small compared to the radius of a normal star (R_0~7 X 10^5 km). On the other hand, its estimated high mass excludes its memberships among neutron stars or black dwarfs. Thus, the source is a compact object but belongs neither to neutron stars nor to black dwarfs. Its most likely classification is thus among the black holes. There are two other likely candidates similar to Cygnus X-1 in our Galaxy, and one in the small Magellan Could. Each of these is a binary system with an X-ray source as one of the components. The two galactic system with X-ray source 2U 1700-37 and 2U 0900-40 respectively, have X-ray spectra similar to that of Cygnes X-1, but their masses have not been determined. The mass of the SMC source, SMC X-1, is estimated to be ~ 10 M_0 from its measured X-ray energy flux.

The observed high radiation flux from radio galaxies, quasars and nuclei of some otherwise normal galaxies calls for its explanation some energy generation mechanism, whose efficiency should be comparable to that of total annihilation of mass. The only such mechanism as currently understood is the gravitation radiation in the intense gravitation field of black holes. So it is not unlikely that these objects contains at their center black holes whose masses may range from $10^5$to 10^9 M_0.

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CHAPTER 2
Black Holes and General Relativity

The basic postulate of general relativity (GR) is that space time is locally Lorentzion at every point provided we use a co-ordinate system suitable for that point. This is Unexceptionable for weak gravitational fields, but if that often-quoted analogy with an elastic solid hold ("space time always tries to straighten itself out"), one has the possibility that this may not hold for sufficiently strong gravitational field wherein space-time maybe globally distorted (e.g. it may not remain hyperbolic). In more modern language, if space- time, including the matter contained in it, is treated as a quantum system the volume State may change. The precise way in which this can happen is not clear. The simplest possibility is that can happen is not clear. The simplest possibility is that of a catastrophic phase transition at some super dense stage during the gravitational collapse leading to the formation of a black hole. We look at the process from a quantum theoretical print of view rather than the purely classical one.

Black holes were at first only a speculation as a result of calculation by Laplace in 1795 when he discussed the classical body with escape velocity greater than that of light but his idea did not attract much attention. Later in 1916 Karl Schwarzschild was able solve the Einstein field equation in the vacuum for uncharged spherical system and his solution is known as the Schwarzschild solution, which implies the simplest black hole type namely the Schwarzschild black hole that is only determined by a single parameter namely its mass M.

The physics of black hole is largely based on Einstein's general theory of relativity (GR), which is a theory of gravitation. Relativity is a geometrical theory because the mathematical study of space-time whether carved or flat is geometry. There is no doubt that GR (including SR as well since SR is just a special solution of GR) is one of the most successful physical theories of mankind (it of course cannot tell everything about the universe.), owing to its predictions that have been confirmed by a number of experimental tests.

In GR physics is expressed in terms of tensors each kind of space-time is assigned with the proper time and space interval which are described by the line element or metric which is an invariant interval for

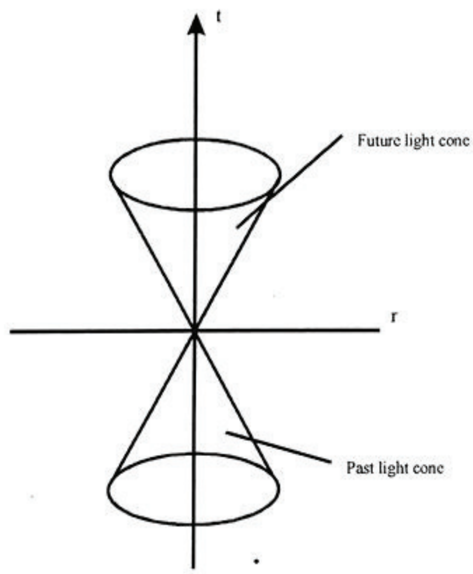

Figure : 2.1:

Figure : 2.1: Light cones in Minkowski spacetime, Any point in the future light cone r = t can be reached by a particle or signal with speed less than c.

$$ds^2 = -dt^2 + dx^2 + dy^2 + ds^2 \text{..} (2.1)$$

on in a tensorial form

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \text{ .... } (2.2)$$

where

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathrm{diag}(-1,1,1,1) \text{ .... } (2.3)$$

and $\eta_{\mu\nu}$ is called the Minkowiski metric tensor or just Minkowski metric for this particular Cartesian coordinates. This metric can turn out to be a positive, negative and zero quantity-it corresponds to a space-like, time-like and light-like metric interval repectively. If we use a general coordinate system,

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu = g_{\mu\nu} dx^\mu dx^\nu \text{ .... } (2.4)$$

The summation sign is suppressed when on use the Einstein summation convention. For example, in spherical polar coordinates we have x = (t, r,0,φ,) thus the metric reads

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \cdots\cdots (2.5)$$

or$g_{\mu v}$ = diag (−1,1,r²,r², sin² 0) which is a flat metric since it can be rewritten using flat coordinates.

When we deal with the curved space-time due to the presence of matter, the Riemann tensor plays a major role as seen in GR. One very important equation in this subject is the Einstein field equation, a tensorial equation which takes the form

$$G_{\mu v}=8_{\pi v}T_{\mu v} \cdots\cdots (2.6)$$

$G_{\mu v}$is Einstein tensor which is symmetric and vanishes when space-time is flat $T_{\mu v}$ is the so called energy-momentum tensor which can be thought of as a source for the gravitational field. It is a divergenceless tensor due to the conservation of energy, namely (v∇µ$T_{\mu v}$ =O. The proportionality constant is$\frac{8\pi G}{c^4}$ since we use the natural units, otherwise it would be$\frac{8\pi G}{c^4}$. Mathematically, the Einstein tensor is given by

$$G_{\mu v}=R_{\mu v} - \frac{1}{2} g_{\mu v} \mathrm{R} \cdots\cdots (2.7)$$

Where $R_{\mu v}$ is called Ricci tensor which is a contraction of the Riemann tensor ($R_{\mu v}$ = $R\gamma_{\mu v}$), and R is a curvature scalar obtained from the Ricci tensor, hence called Ricci scallar. The full form of the Einstein euqation has an extra term owing to the Cosmological constant (A) which has been found recently to be an extremely tiny number but non-zero. It reads

$$G_{\mu v} \ + \ g_{\mu v} = 8\pi T_{\mu v} \cdots\cdots (2.8)$$

The significance of the cosmological constant is involved mostly in the context of cosmology in which one studies the fate of the universe. The cosmological constant will appear again when we discuss the BTZ black hole in Section 2.5.

2.1 Kerr Metric :-

It is known that a black hole can be entirely described through the use of only three parameters the mass M, angular momentum J (often expressed in terms of units of a mass a J/M) and residual charge Q. Though the mass of observed black holes ranges over several orders of magnitude, it is believed that a tends to be nonzero and significant, and that the residual charge Q is nearly identically zero. Thus, we seek a metric for a spinning black hole with no charge. This is known as the Kerr metric, and within the equatorial plane in Boyer-Lindquist coordinates, it has the following from.

$$\mathrm{d}\tau^2 = \left(1 - \frac{2M}{R}\right) dt^2 + \frac{4Ma}{r} \mathrm{dtd}\phi$$
$$- \frac{dr^2}{1 - \frac{2M}{R} + \frac{a^2}{r^2}} - \left(1 + \frac{a^2}{r^2} + \frac{2Ma^2}{r^3}\right) r^2 \mathrm{d}\phi^2 \cdots\cdots (2.9)$$

In some of the analysis in this paper, we will consider only maximally spinning black holes (a = 1), and introduction a reduced circumference R for convenience. The metric in this case is

$$d\tau^2 = \left(1 - \frac{2M}{R}\right) dt^2 + \frac{4M^2}{r}\mathrm{dtd}\phi$$

$$-\frac{dr^2}{\left(1 - \frac{M}{Rr}\right)^2} - R^2\mathrm{d}\phi^2 \cdots\cdots\cdots\cdots (2.10)$$

$$R^2 = r^2 + M^2 + \frac{2M}{R^3} \cdots\cdots\cdots\cdots (2.11)$$

A spinning black hole has several noteworthy properties. First, part of the measured mass M is actually stored as rotational energy, and is in theory available for extraction, since it is not mass that the passed the event horizon. Second, there exist serval critical values of r in the metric. Before the normal event horizon, there exists a static limit a surface past which not even light can resist being dragged along with the black hole.

2.2 Penrose Process: -

In 1969, Roger penrose, a post-doctoral student at Cambridge, published a paper on the role of general relativity in gravitation collapse [25]. In it, he presented a mechanism by which the rotational energy of a black hole might be extracted.

Penrose noted that the while the process was not "practical", there might yet be some "indirect relevance to astrophysical situations". Crucial to his argument is the existence of a region of negative total energy in the vicinity of a spinning black hole.

A "Penrose Process"

We now show that regions of negative energy can be used to extract energy from a black hole. Boiled down to its essentials, the Penrose Process in simple:

1. Two Practical of equal mass are at rest at infinity.
2. Both are moved into a region of negative energy in vicinity of the black hole.
3. One practical is captured, decreasing the mass of the hole.
4. The other practical escapes to infinity, with more energy than the initial energy of both particles.

Evidently, the black hole has imparted some of its rotational energy to the practical.

At this point, the problem becomes optimizing the parameters of a generic orbit to find the maximum energy out to energy-in ratio. In 1983, Chandrasekhar [26] showed that this efficiency was about 20.7%. We make a similar calculation using the more idealized process presented in [27].

2.3 Kerr-Newman Penrose Process

We have freely used two of the parameters available to us to describe the black hole - spin J and mass M. In 1985, Bhat, et. al., a group of researchers at the University of Pune in India, studied how the negative energy region was affected by the presence of a residual net charge [28].

Justification of the relevance of this analysis is not trivial, since astrophysical objects are not in general known to carry significant residual charge. However, because of the scale differences at which electromagnetic and gravitational effects become important, even a relatively small residual charge could have a dramatic effect. Specifically, Bhat et. Al. set $Q/M \ll 1$, assumed it could be left out of the metric of the black hole, but that added the Lorentz force to the equations of motion.

The Kerr metric is the one actually used, and an electrostatic term is artificially added to the expression for energy. In particular, this can be realized by adding the electromagnetic energy potential $V(r) = -qQ/r$ to the energy quantity for a particle with charge q.

2.4 Black Hole Metrics: -

The geometry of a spherically symmetric vacuum, i.e. vacuum spacetime outside the spherical black hole is the Schwarzschild geometry described in terms of the Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \text{ .... } (2.12)$$

where $d\Omega^2 = d\theta^2 sin^2\theta d\varphi^2$. The Schwarzschild metric is indeed a gravitational field and seems to have a singularity at the surface where r = 2M due to its coordinates in which space and time change their meanings. To get an idea of the magnitude of the Schwarzschild radius we note that for the Earth it is 0.1 cm and for the star of the size of the Sun, it is about 3 kilometers. The singularity at r = 2M is however removable by choosing coordinates cleverly, an alternative to that is the Kruskal-Szekers coordinates found in 1960, which represent the space-time more properly. The surface of the black hole is entitled an "event horizon" for the fact that nothing can be seen beyond it. Only region on and outside the black hole's surface, r > 2M is observationally relevant. The Schwarzschild metric is necessarily.

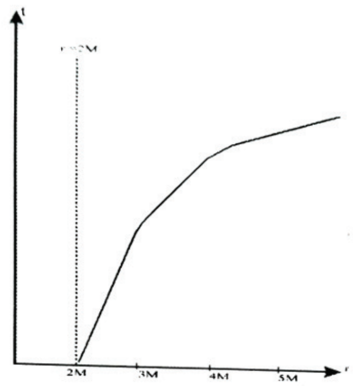

Figure 2.2

Figure 2.2: A trajectory of a practical or a single approaching the Schwarzschild black hole as represented in the Minkowski spacetime. The surface r = 2M is the event horizon, nothing inside surface is able to escape. This trajectory is given by solving the Schwarzschild metric $ds^2 = 0$ (null geodesic) with $d\Omega^2 = 0$. Asymptotically flat, that is, for large r,

$$ds^2 \approx -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 + \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \text{ .... } (2.13)$$

and in addition to the Eq. (2.13) one can show that the Newtonian gravity is merely a limiting case of GR.

So far, we have seen that the Schwarzschild black hole depends only on the mass. This simple black hole is, astronomically, a collapse of an uncharged and non-rotating star with spherical symmetry. The gravitational collapse of a non-spherical star with non-zero net charge produces a somewhat different black hole which can be characterized by mass M, intrinsic angular momentum or spin J and electric charge Q. It is found that the structure of a black hole is determined uniquely by just three parameters, i.e. M, J and Q once it is in the final state. In this way it can be said that "black holes have no hair" [10]. The black hole in the final state with only M and Q, has a gravitational field given by the Reissiner-Nordstrom metric which takes the form

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad \cdots\cdots\cdots\cdots (2.14)$$

Where M and Q are the total mass and charge respectively as measured by a distant observer. For the rotating uncharged black hole, (namely the black hole is characterized only by M and J) its geometry is given by the Kerr metric (usually represented

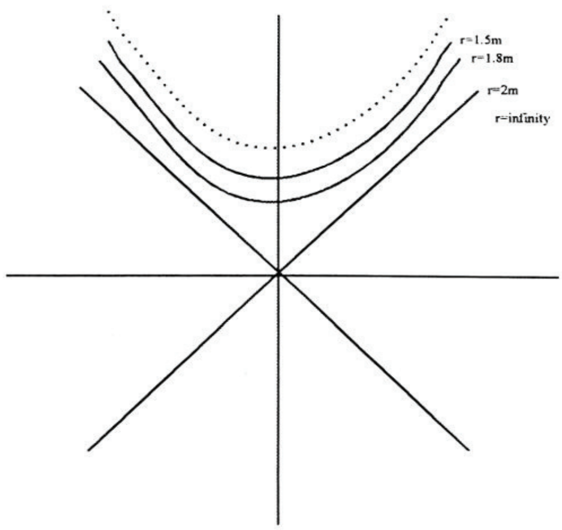

Figure 2.3:

Figure 2.3: The Schwarzschild blackhole in the Kruskal-Szekeres coordinates have no "coordinate singularity" hence represents the real spacetime where r=0 is the singularity which is the broken thick line in the figure.

in the Boyer-Lindquist coordinates) using a J/M. The significance ofthe vlaue of a plays a role when the extremal case is considered, i.e.

$\frac{a}{M} = 0 \Rightarrow$ There is no spin, hence reduced to the Schwarzschild case.

$\frac{\boldsymbol{a}}{M} = 0 \Rightarrow$ Extremal Kerr black hole is reached.

The Kerr metric takes the form:

$$ds^2 = -\frac{\Delta - a^2 sin^2\theta}{\rho^2} dt^2 + 2a\frac{2Mrsin^2\theta}{\rho^2} dtd\varphi$$

$$+\frac{\left(r^2+a^2\right)^2 - a^2\Delta sin^2\theta}{\rho^2} sin^2\theta d\varphi^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \quad (2.15)$$

Where

$$\Delta = r^2 - 2Mr + a^2 \quad (2.16)$$

$$\rho^2 = r^2 + a^2 cos^2 0 \quad (2.17)$$

Its event horizons are (assuming that $a^2 < M^2$

$$r\pm = M \pm \sqrt{M^2 - a^2} \quad (2.18)$$

The Kerr metric or Kerr solution is stationary and axially symmetric and has double surface, i.e. outer and inner surfaces. In between the event horizon and the static limit lies the so-called Ergosphere inside which nothing can remain stationary.

For the charged rotating black hole (named Kerr-Newman black hole), its geometry is given by the Kerr-Newman metric which is in the same from as Eq (2.15).

but with

$$\Delta = r^2 - Mr + a^2 + Q^2 \quad (2.19)$$

The event horizons of the Kerr-Newman black hole are

$$r \pm = M \pm \sqrt{M^2 - Q^2 - a^2} \quad (2.20)$$

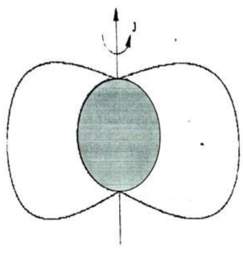

Figure 2.4:

Figure 2.4: A rough sketch of the Kerr black hole which is surrounded by an ergo sphere. The ergo sphere is a region inside which nothing can remain stationary. The angular momentum of the Kerr black hole is denoted by J.

for $a^2 < M^2+Q^2$. The extremal Kerr-Newman black hole is reached when $a^2 = M^2+Q^2$.

It is notable that one would hardly observe charged black holes in nature because the black hole is already discharged when it is in stationary state. The discharging process takes place very rapidly. The time it takes to become stationary is called the characteristic time which is approximately $10^{-5}\frac{M}{M_0}$second. Therefore, it is obvious that the black hole of 100000Mo only requires approximately 1 second reach its stationary state.

2.5 The BTZ Black Hole: The (2+1)-dimensional black hole

Banados, Teitelboim and Zenelli discovered, in 1992, the black hole solution of Einstein's equation with a negative cosmological constant, in 2+1 dimensions and without couplings to matter [29,30]. This discovery was rather surprising as there was no speculations that there would exist a black hole solution in 2+1 dimensions at that time. The BTZ black hole is known as a simple toy model for a number of studies including the string and supergravity theory. The cosmological constant, A is written as $-l^{-2}$ in the BTZ black hole's metric which reads:

$$ds^2 = -N^2(r)dt^2 + N^{-2}(r)dr^2 + r^2(N^{\varphi}dt + d\varphi)^2 \quad \cdots\cdots\cdots\cdots\cdots (2.21)$$

$$N^2(r) = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, N^{\varphi} = -\frac{J}{2r^2} \quad \cdots\cdots\cdots\cdots\cdots (2.22)$$

with$-\infty < t < \infty$, $0 < r < \infty$ and $0 \leq \varphi \leq 2\pi$. $N^2(r)$ and $N^{\varphi}$are the squared lapse and angular shift respectively.

The event horizons can be obtained from $N^2(r) = 0$ and takes the from:

$$r\pm = l\sqrt{\frac{M}{2}(1 \pm \Delta)} \quad \cdots\cdots\cdots\cdots\cdots (2.23)$$

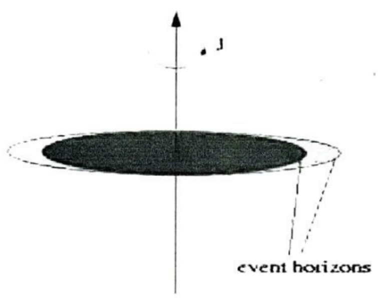

Figure 2.5

Figure 2.5: The 2+1 dimensional BTZ black hole. This black hole can be visualized as a circular disc with spin J. Its event horizon depends on its mass.

where

$$\Delta = \sqrt{1 - \left(\frac{J}{ML}\right)^2} \quad \cdots\cdots\cdots\cdots\cdots (2.24)$$

with imposed conditions that

$M > 0$ and $|J| \leq Ml$ ···2.25

In the extremal case J = $[Ml]$, the two event horizons coincide. Note that l is the radius of curvature which provides the length scale in order to have dimensions less mass. If one lets l grows very large the black hole exterior is pushed away to infinity and one will be left with inside [29]. The BTZ black hole is similar to its 3+1 counterpart, the Kerr solution. The BTZ black hole has an crgosphere namely $r_{erg} = l\sqrt{M}$and an upper bound in angular momentum for any given mass.

The space-time geometry of the black hole is one of constant negative curvatures, so it is locally that of anti-de Sitter (adS) space. The BTZ black hole can only differ from the adS in its global properties [30]. The BTZ black hole is however different from its counterpart by the fact it has a positive heat capacity instead of the negative heat capacity. Therefore, its thermodynamics properties are well defined and simple.

If we, for the sake of simplicity set A =-1 (hence $l = 1$) for the spinless BTZ black hole its metric is then reduced to

$$ds^2=(-M+r^2)\, dt^2 + (-M+r^2)^{-1}\, dr^2+r^2 d\varphi^2 \cdots\cdots\cdots \quad (2.26)$$

The metric above is singular when r = $\sqrt{M}$, which is an event horizon of the spinless BTZ black hole.

2.6 Entropy of a black hole

Black holes confront us with fundamental problem: what happens to the information's when a particle falls inside a black hole?

Remember that only three parameters are required to fully describe a black hole: its mass, its electrical charge and its angular momentum.

But, in order to describe a physical system we need other information, especially entropy, which is a measurement of its disorder.

Losing entropy by falling into a black hole is violation of the second principle of thermodynamics. This law states that entropy is an always increasing function in a closed system and the universe is a closed system, as nothing can escape it.

In 1972, Stephen Hawking showed that the area of the horizon of a black hole cannot decrease.

(note: what we call here the area of the black hole is in fact the surface defined by its horizon)

In particular in the case of the merging of two black holes the area of the final horizon can't be less than the sum of the areas of their initial horizons.

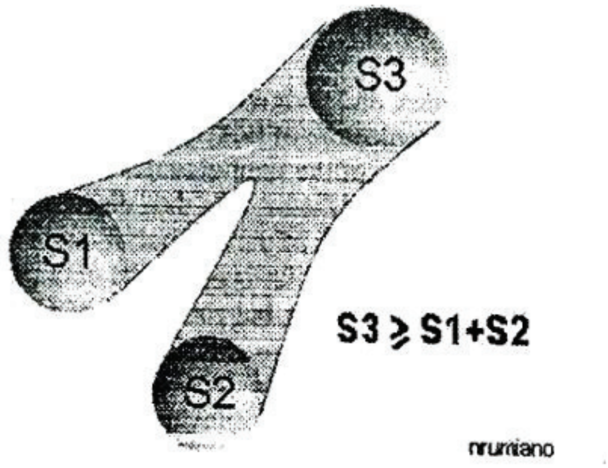

Fig. 2.6

Hence, Jacob Bkenstein identified the area of the black hole which can never decrease with entropy: if this area represents a measure of the entropy of the black hole the second principle is no longer violated.

The entropy of a black hole is expressed as where A is the area of the black hole K the Boltzmann's constant, h the Planck's constant, c the speed of light and G the gravitational constant. Fine but a new problem arises: if the black hole has an entropy, it must have a temperature too.

Everybody which has a temperature is able to radiate energy according to a spectrum which must correspond to its temperature. But in its classical definition nothing can escape a black hole.

2.7 Black holes and Thermodynamics

Black holes are found to be intimately related to ordinary thermodynamics in the sense that their mechanical laws are seemingly identical to the laws of the thermodynamics.

Thermodynamic system	Black hole
Temperature, T	Surface gravity, k
Energy, E	Black hole's mass, M
Entropy, S	Area of event horizon, A

Table 2.1: Analogy between thermodynamic parameters and black hole's parameters.

In 1971 Stephen Hawking [9] stated that the area, A of the event horizon of a black hole can never decrease (but can remain constant) in any process:

$$\Delta A \geq 0 \quad (2.27)$$

The area of the event horizon increases when (1) mass increases and (2) spin decreases. It was later noted by Bekenstein [8] that this result is analogous to the statement of the ordinary second law of thermodynamics, namely that the total entropy S of a closed never decreases in any process.

$$\Delta S \geq 0 \cdots\cdots (2.28)$$

With these arguments it is legitimate to establish the laws of black hole mechanics in parallel to the laws of ordinary thermodynamics by using parameters of the black hole (see Table 2.1) as above.

Zeroth law: The event horizon is described by a quantity, K the surface gravity which is constant over the event horizon. The surface gravity is related to the physical temperature of the black hole (Hawking temperature) by

$$T_H = \frac{K}{2\pi} \cdots\cdots (2.29)$$

For the special case of the Schwarzschild black hole, where k=1/(4GM), the Hawking temperature becomes:

$$T_H = \frac{\hbar}{8\pi G K_B M} \approx 6.2 \times 10^{-8} \frac{M_0}{M} K \cdots\cdots (2.30)$$

So, this is utterly negligible for solar-mass black holes the black hole absorbs much more from the microwave background radiation than it radiates itself. In the case of the rotating "Kerr" black hole, the Hawking temperature is reduced by the rotation, explicitly

$$T_H = \frac{\hbar k}{2\pi K_B} = 2\left(1 + \frac{M}{\sqrt{M^2 - a^2}}\right)^{-1} \frac{\hbar}{8\pi M K_B} < \frac{\hbar}{8\pi M K_B} \cdots\cdots (2.31)$$

where a = J / M For the charged non-rotating "Reissner-Nordstrom" black hole, one has

$$T_H = \frac{\hbar k}{2\pi K_B} = \left(1 - \frac{Q^4}{r_+^4}\right)^{-1} \frac{\hbar}{8\pi M K_B} < \frac{\hbar}{8\pi M K_B} \cdots\cdots (2.32)$$

Thus, electric charge also reduces the Hawking temperature. As a conclusive remarks one can safely say that the Hawking radiation plays no role in the case of large-sized black holes. The only type of black hole where one can hope to observe such the radiation is the so-called mini black hole, which is believed to have existed in the primordial stage of the Universe.

First Law: This law deals with the mass (energy) change, dM when a black hole switches from one stationary state to another.

$$\left.\begin{array}{l} dM = \left(\frac{k}{8T}\right) dA + \text{“work done”} \\ \text{or} \\ dM = T_H dS_{bh} + \text{“work done”} \end{array}\right\} \cdots\cdots (2.33)$$

It is readily seen that the above equations are analogous to the first law of thermodynamics, i.e.

$$dE = TdS + \text{“work terms”} \cdots\cdots (2.34)$$

And the entropy of the black hole is thus represented by a quarter of the area of the event horizon, that is

$$S_{bh} = \frac{A}{4} \cdots\cdots (2.35)$$

The factor 1/4 was indeed found by Hawking [8] based on the applications of the quantum field theory to the black holes which shows that they will absorb and emit particles as if they were thermal bodies with the Hawking temperature, T_H. S_{bh} is sometimes called Bekenstein-Hawking entropy in order to honor their discoveries. The "work terms" are given differently depending on the type of the black holes. For the Kerr-Newman black hole family, the first law would be

$$dM = \left(\frac{k}{8\pi}\right)dA + \Omega\, dJ + \Phi dQ \quad \text{(2.36)}$$

where Ω is the angular velocity of the hole and Φ is the electric potential which are defined by

$$\Omega = \frac{\partial M}{\partial J} \quad \text{(2.37)}$$

$$\Phi = \frac{\partial M}{\partial Q} \quad \text{(2.38)}$$

Second law: In any classical process, the area of the event horizon does not decrease.

$$dA \geq 1 \quad \text{(2.39)}$$

nor does the black hole's entropy, S_{bh}. The second law of black hole mechanics can, however, be violated if the quantum effect is taken into account, namely that the area of the event horizon can be reduced via Hawking radiation. Note that the proof of this depends on the Cosmic censorship conjecture. It is essential.

Law	Thermodynamic system	Black hole
Zeroth law First law Second law Third Law	T constant on a body in thermal equilibrium dE=t dS-pdV+μdN dS> 0 T= 0 cannot be reached	k constant over a black hole's event horizon $dM=\frac{k}{8\pi}dA + \Omega\, dj + \Phi\, dQ$ $dA \geq 0$ $k = 0$ cannot be reached

Table 2.2

Table 2.2: Analogy between the laws of thermodynamic and the laws of black-hole mechanics.

That the black hole radiation is thermal in nature therefore generates a rise in entropy in the surrounding region. The generalized entropy, S' was introduced by Bekenstein [3,31,32] to account for this sort of entropy. It is defined as the sum of the black hole's entropy, S_{bh} and the entropy of the surrounding matter, S_m

$$S^l = S_{bh} + S_m \quad \text{(2.40)}$$

This statement is known as the Generalized Second Law (GSL):

$$\Delta S^{l} \geq 0 \cdots\cdots \quad (2.41)$$

The ordinary second law seems to fail when the matter is dropped into a black hole because according to classical GR, the matter will disappear into a spacetime singularity, in this manner the total entropy of the universe decreases as there is no compensation for the lost entropy. The virtue of the GSL keeps the law of entropy valid as the total entropy of the universe still increases when that matter falls into the back hole.

It is natural to question the magnitude of the entropy of the black hole when expressed dimension fully, the bekenstein-hawking entropy reads:

$$S_{bh} = \frac{kbA}{4Gh} \cdots\cdots \quad (2.42)$$

for the Schwarzschild black hole, this yields [11]

$$S_{bh} = \frac{k_B \pi R_0^2}{Gh} \cdots\cdots \quad (2.43)$$

Numerically, the entropy of the sum is $S_0 \approx 10^{57} K_B$ whereas a solar-mass black hole has an entropy $10^{77} K_B$ which is 20 orders of magnitude larger!

Third law: The limited k = 0 cannot be reached within a finite time, in other words it is not possible how many processes we do, we will never reach the limit k=0. However, the extremal black, for example the Kerr black hole in which a/M=1, do have k=0 thus zero temperature (absolute zero) but non-zero entropy. To actually reduce the surface gravity to zero is merely an idealized case because it is forbidden by the Cosmic censor-ship conjecture.

As far as physical theories are concerned, the laws of black hole mechanics have only been phenomenological thermodynamical laws [11], thereby a number of open questions can be raised, such a [33,34,35].

* Is S_{bh} real or subjective?

* Where does it appear on or near the horizon or deep in the hole?

* At what stage in the black hole's evolution is it created-immediately upon formation by gravitation collapse, or only gradually over the long course of evolution?

* What is the dynamical that makes S_{bh} a universal function, independent of the hole's past history or detailed internal condition?

When the quantum effects are taken into account, one can ask: Can S_{bh} be derived from quantum mechanical considerations?

Due to the effect of Hawking radiation (black hole evaporation) what happens to S_{bh} after the black hole has evaporated? will all the information disappear after the evaporation?

These questions are somewhat embarrassing, because we do not Know with our present knowledge how to answer them precisely. Nevertheless, it is hoped that success in modern theory of gravity, e.g., the quantum, gravity or the string theory would be the key to answer-if not all-some of these open questions.

The many formal analogies between black-hole physics and thermodynamics the most striking of which is that between Hawkins's classical theorem that "black-hole surface area cannot decrease" and the second law of thermodynamics that "entropy cannot decrease," make the attempt at a physical synthesis of the two disciplines worthwhile. Essential to the success of such a program is a version of the second law relevant to system containing both matter and black holes. We conjectured that this generalized second law (GSL) must take the from “black-hole entropy S_{bh} pulls common (ordinary) entropy exterior to black holes S_c never decreases”.

$$\Delta Sg = \Delta S_{bh} + \Delta S_c \geq 0$$

Where S_{bh} is given in terms of black-holes surface area A by

$$S_{bh} = \eta\, h^{-1} A$$

(Units with G = c = k = 1 are assumed). Here h is the Planck area ($2.6 \times 10^{-66} cm^2$) and η is a numerical constant of order unity to be determined by independent arguments.

The GSL is designed to replace the ordinary second law, which is transcended in the presence of a black hole because the common entropy interior to the holes is unobservable by exterior observers. On the other hand, the GSL makes a stronger statement then the area theorem. The theorem requires only that A not decrease. The GSL demands that if exterior entropy is lost into black holes. An increase sufficiently for the associated increase in S_{bh} to, at least, compensate for the decrease in S_c. That this actually happens has been verified for the case of the infill of a small entropy package into a generic stationary black hole.

The area theorem based as it is on a classical energy condition it’s expected to be violated by some quantum processes. By contrast the GSL being an intrinsically quantum law could be expected to fare better. And in deed in the astonishing quantum process of spontaneous radiation by a Kerr black hole discovered by Hawking, the area theorem is flagrantly violated but the increase in exterior entropy due to the radiation is expected to suffice to uphold the GSL. This confirmation of the validity of the GSL for a process not even dreamt of at time of its inception is striking evidence of the versatility of the law and of the physical meaning-fulness of the concept of black-hole entropy which underpins it. In view of these developments, the time appears ripe for considering black-hole thermodynamics from a statistical viewpoint with the ultimate purpose of clarifying the statistical significance of black-hole entropy.

Traditional methods of statistical thermodynamics to treat serval interconnected problems in black-hole thermodynamics. Using Jayne's information-theory approach for computing entropy, we show explicitly that the GSL (in a statistically averaged from) is respected by the process of spontaneous radiation by an isolated Kerr black hole, as suggested by Hawking. By the same approach the GSL in averaged from is also respected in the case of a Schwarzschild hole immersed in a blackbody radiation bath regardless of the latter's temperature.

We generalize the principle of maximum entropy to embrace black-hole entropy. As an application we determine the probability distribution for radiation in thermodynamics equilibrium with a Kerr hole and the associated probability distribution for the various Kerr solution states off definite mass. The same result follows directly from a statistical interpretation of black hole entropy as the natural logarithm of the number of possible interior configurations of the black hole which are compatible with the exterior state in questions.

2.8: Interior Configurations of a Black Hole

Statistically, thermal entropy can be regarded as the natural logarithm of the number of possible distinct microscopic configurations of the system compatible with the microstate in questions. In introducing the black-hole entropy, we suggested by analogy that it may be interpreted as the natural logarithm of the number of possible distinct interior configuration of a black hole compatible with the exterior black hole state (for example, a Keer solution state) in question. We shall now find support for this conjecture in that it leads very directly to the result we obtained in the preceding section.

We consider a Kerr hole in equilibrium with radiation in non super radiant modes within a container. Each distinct "microstate" of the system consists of a radiation state specified by occupation numbers and an interior black-hole configuration. We make the traditional postulate that all microstate of a system in equilibrium are equally probable (equal a priori probabilities). It follows that the probability of a radiation state regardless of which interior configuration it goes with must be proportional to the number of configurations compatible with it, or, equivalently, compatible with the Kerr solution state associated with that radiation state. By our conjecture this number is just the exponential of the black-hole entropy of the Kerr solution state. We see that indeed a blackhole in a given Kerr solution state behaves as if it can be in any of a number of equally probable interior configurations, with the logarithm of this number giving the black-hole entropy. An interesting way to look at these interior configurations in a particular model of collapse has been given by Gerlach.

It is interesting to contrast the approaches of generalized maximum-entropy principle for equilibrium systems and the present section, which both lead to the same result. In generalized maximum entropy principle for equilibrium system, we use a principle motivated by the GSL. It treats the black hole as a black box, and it does not attempt to give an interpretation to S_{bh}, but simply takes it as a property of the exterior state of the black hole. In the present section the GSL does not come in in any way, but S_{bh} is given a statistical interpretation in terms of the black-hole interior. On two approaches could be more dissimilar, and the agreement between their results speaks for the self-consistency of the ideas we have made to use.

2.9: Jaynes's Maximum-Uncertainty Principle for Black Holes

The GMEP is use only for equilibrium systems. Something else is needed for nonequilibrium situations involving black holes; for example, one problem is determining the distribution P_{MLQ} for a Kerr hole radiation in isolation. We may here be guided by Jaynes's maximum-uncertainty principle, already applied which states that the best probability distribution one can assign to the states i of a system is the one which maximizes the information theoretical (or entropy).

$$S=-\sum_i p_i^{Inp.}\text{i},$$

Subject to the known information. This principle is supposed to apply to nonequilibrium systems, as well as to equilibrium ones. For the latter it is identical to the usual maximum-entropy principle. Here we would like to formulate a Jaynes-type principle for a Kerr hole by itself.

The states i of which the principle speaks are the elementary states of the system. For a black hole these would be the interior configurations and not Kerr solution states, which are classes of interior configuration. Nevertheless, since only the Kerr solution states are externally observable, it is advantageous to re-express S in terms of the P_{MLQ}. It seems reasonable to assume that all interior configuration for a given Kerr solution state is equally probable-they all share the same dynamical quantities M, L, and Q and consequently are totally equivalent to an exterior observer.

So, for given M, L, and Q

$Pi = P_{MLQ}/N_{MLQ}$,

Where N_{MLQ} is the number of interior configurations corresponding to the given Kerr solution state.

*****

CHAPTER 3
Thermodynamics Fluctuation Theory

Thermodynamics is an old subject in physics and it is applicable to a wide variety class of systems. The ordinary laws of thermodynamics are not fundamental in their own right, but are laws that arise from the microscopic properties of the system. The utility of the thermodynamic laws is based on the fact that has a universal validity, at least for most systems.

3.1 The role of entropy

Entropy is, in a sense, a measure of the disorder of a system. This quantity was first introduced by R. Clausius in 1850 as the amount of heat reversibly exchanged at a temperature T. Entropy undoubtedly plays a major role in thermodynamics and statistical mechanics. It is also the most characteristic extensive parameter in thermodynamics, namely when it is expressed in terms of other extensive parameters-it basically tells us the physics that underlines the system. Entropy is a part of the first and the second law of thermodynamics directly-it enters the first law to complete the differential representation of the internal energy, namely

$$dU = TdS - pdV + \mu dN \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.\ 1)$$

We can also express the entropy as a thermodynamics potential as

$$dU = \frac{1}{T}dU + \frac{P}{T}dV - \frac{\mu}{T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

Which is just the differential form of the entropy. In this view, if the dependence of the entropy S (U, V, N) on the variables U, V, N is known, then complete knowledge of all the thermodynamics parameters is obtained. Furthermore, the entropy tells us that for isolated systems (where $dQ_{reversssble}=0$) in equilibrium

$$dS = 0 \Leftrightarrow S = S_{maxitmum} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

and for irreversible process

$$dS > 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

so in words it says that the state of equilibrium is defined as the state of maximum entropy. Now as an example we have the entropy of the ideal gas a [36]

$$S(N, T, p) = Nk_B\left\{so\left(T_{o,po}\right) + 1n\left[\left(\frac{T}{T_0}\right)^{5/2}\left(\frac{po}{p}\right)\right]\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

Where $s_o(T_o, p_o)$ is an arbitrary dimensionless function of the state (T_o, P_o) which is the result of the integration since Eq. (3.34) is derived from the second law of thermodynamics when we use $PV=Nk_BT$ *and* $U=\frac{3}{2}Nk_BT$. It is obvious that Eq. (3.34) gives us full information about the ideal gas. The entropy of the ideal gas can also be expressed in terms of the other extensive parameters, i.e. S (N, V, U) and it reads [37]

$$S(N, V, U) = Nk_B\left\{so(N_0, V_0, U_0) + 1n\left[\left(\frac{N_0}{N}\right)^{\frac{5}{2}}\left(\frac{U}{U_0}\right)^{3/2}\left(\frac{V}{V_0}\right)\right]\right\} \cdots\cdots\cdots\cdots (3.6)$$

From this equation, all equations of state of the ideal gas can be obtained by partial differentiation. Therefore by differentiating Eq. (3.6) with respect to the internal energy yields

$$\left(\frac{\partial S}{\partial U}\right)_{N,V} = \frac{1}{T} = \frac{3}{2}Nk\frac{1}{U} \Rightarrow U = \frac{3}{2}NkT \cdots\cdots\cdots\cdots (3.7)$$

Whereas differentiating it with respect to the volume we get the equation of state:

$$\left(\frac{\partial S}{\partial V}\right)_{N,U} = \frac{p}{T} = Nk\frac{1}{V} \Rightarrow pV = NkT \cdots\cdots\cdots\cdots (3.8)$$

From statistical mechanics the entropy of the system is given by natural logarithm of the number of microscopic state[1] Ω which reads (with the Boltzmann's constant being unity):

$$S = \text{In}\ \Omega \ldots\ldots\ldots\ldots (3.9)$$

The microstate Ω will be a function of the microstate, i.e., Ω (U, V, N) when the entropy is a function of the variables. Eq. (3.9) is very important for it provides the basic connection between macroscopic thermodynamics (entropy) and statical microscopic physics (number of microscopic states). Notice that S=O when Ω = 1, thus there is only one exact microstate, hence no disorder and no entropy is created.

3.2 Fluctuations

Physical quantities which describe a macroscopic body in equilibrium are, almost always, close to their mean values. However, there are certain small deviations from the mean values, which is the natural behavior of the system. These deviations are known as thermodynamic fluctuations. The problem that arises is to find the probability distribution of these deviations.

When Eq. (3.9) is inverted one gets which is the starting point of thermodynamic fluctuation theory indeed as it was first done by Einstein. We, however, preferably denote P as the probability distribution, i.e. $P \propto e^s$, we can Taylor expand the entropy about the fluctuation quantity x only up to the second order as

$$S(x) = S\ (0) - \frac{1}{2}\beta x^2 \ldots\ldots\ldots\ldots (3.11)$$

Where$\beta = \frac{\partial^2 S}{\partial x_2}|0$ and S (0) = 0 since the entropy S has a maximum for $x = 0$ as $\frac{\partial S}{\partial x}$=0. Substituting Eq. (3.11) into Eq. (3.10) yields:

$$P(x) = Ae - \frac{1}{2}\beta x^2 \ldots\ldots\ldots\ldots (3.12)$$

In differential form it reads:

$$P(x) = dx = Ae - \frac{1}{2}\beta x^2 dx \ldots\ldots\ldots\ldots (3.13)$$

The constant A is given by the normalization condition that $\int P(x)\,dx = 1$.

The integration limit is over all space, i.e., from $-\infty\ to\ \infty$. This constant is found to be $\sqrt{\beta/2\pi}$ by Gaussian integration formula. Thus, the probability distribution of the various values of the fluctuation reads:

$$P(x)=\sqrt{\frac{\beta}{2\pi}e-\frac{1}{2}\beta x^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.14)$$

This probability distribution is categorized as a Gaussian distribution. It reaches a maximum when $x=0$ and decreases rapidly and symmetrically as $|x|$ increases.

The mean squared fluctuation is defined as

$$\langle x^2\rangle=\int dxP(x)x^2=\frac{1}{\beta}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.15)$$

We can then write the Gaussian distribution as

$$P(x)dr=\frac{1}{\sqrt{2\pi(x)^2}}exp\left(-\frac{x^2}{2\langle x^2\rangle}\right)dx\ldots\ldots\ldots\ldots\ldots\ldots (3.16)$$

It is then readily seen that, the smaller the $\langle x^2\rangle$, the sharper the maximum of P(x), which is the characteristics of the Gaussian distribution. Let us now consider the Gaussian distribution for more than one variable, namely we will determine simultaneous deviation of several thermodynamic quantities from their mean values. We define the entropy S (x_1, ….., x_n) as a function of the quantities of a simultaneous deviation, and Taylor expand S in the same manner as done in Eq. (3.11) to the second order, thus

$$S-S_0=-\frac{1}{2}\sum_{ij=1}^{p}\beta ijxixj\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.17)$$

note that $\beta_{ij}=\beta_{ji}$. For simplicity we will omit the summation sign, we thus write

$$S-S_0=-\frac{1}{2}\beta ijxixj\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.18)$$

and the probability takes the form

$$P=Aexp\left(-\frac{1}{2}\ \beta ijxixj\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.19)$$

A is the normalization constant which is determined from the normalization condition namely,

$$\int dx_1\int dx_2\ldots\int dx_nP(x_1\ldots .x_n)=1\ldots\ldots\ldots\ldots (3.20)$$

After some algebraic manipulation (see [38], we get

$$A=\frac{\sqrt{\beta}}{(2\pi)^{\pi/2}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.21)$$

Therefore, the desired form of the Gaussian distribution for more than one variable reads

$$P=\frac{\sqrt{\beta}}{(2\pi)^{\pi/2}}exp\left(-\frac{1}{2}\ \beta ijxixj\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.22)$$

Where

$$\beta ij = -\frac{\partial^2 s}{\partial xi \partial xj}, \beta = |\beta ij| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.23)}$$

3.3 Thermodynamics and Geometry

Ruppeiner [39] in 1979 proposed a new way to study thermodynamics by using a Riemannian geometrical model, in other words, he claims that thermodynamic systems can be represented by Riemannian geometry and certain statistical properties can be derived from the model. This geometrical model is based on the inclusion of the theory of fluctuation into the axioms of equilibrium thermodynamics, namely there exist equilibrium states which can be represented by points in two-dimensional surface (manifold) and the distance between this equilibrium state is related to the fluctuation between them. This concept is associated to probabilities, i.e., the less probabilities, i.e., the less probable a fluctuation between states, the further apart they are. This can be recognized if one considers βij in the distance formula (line element) between the two equilibrium states

$$ds^2 = \sum_{ij} \beta ij dx^{id}\, dxj\ = \beta ij dx^i dx^j \ldots\ldots\ldots\ldots\ldots\ldots \text{(3.24)}$$

Where the matrix of coefficients βij is the metric tensor, which is symmetric. We call a manifold with a rule for distance in the form of Eq.(3.24) a Riemannian manifold, if we now define

$$x_i = \frac{\partial s}{\partial x^i} = \beta ij x_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.25)}$$

We will find that [38]

$$\langle x^i x^j \rangle = \beta ij = \langle x^i x^j \rangle \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.26)}$$

Which is the second moment for the fluctuation or the pair correlation function. We refer the variables x^i as thermodynamically conjugate to x_i and we will assign S_{ij} to be the metric tensor instead of β_{ij} the probability β_{ij} distribution can then be rewritten as

$$P(x) = \frac{\sqrt{S}}{(2\pi)^{\pi/2}} exp\left(-\frac{1}{2} S_{ij} X^i X^j\right) \ldots\ldots\ldots\ldots\ldots\ldots \text{(3.27)}$$

where

$$S_{ij} = \frac{\partial^2 s}{\partial x^i \partial x^j} S = \left|Si_j\right| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.28)}$$

It is not entirely clear from the outset how this can be used to describe thermodynamic system, but if we limit ourselves to the coordinates which are extensive parameters in ordinary thermodynamics, we will be able to obtain some thermodynamic properties. the following sections will discuss this.

3.3.1 Ruppeiner metrics

The Ruppeiner metric is defined as the second derivatives of the entropy of the system with respect to the internal energy and other extensive charge (thermodynamic parameters), namely $Q^i=(M, N^a)$, it reads:

$$S_{ij} = \frac{\partial^2 s}{\partial Q^i \partial Q^j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.29)}$$

N^a is a conserved quantity and a = 1 ,...,n. An interesting fact about this metric is that its inverse form is

$$S_{ij} = \frac{\partial^2 r}{\partial \beta_i \partial \beta_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.30)}$$

where $\beta_i = \left(\frac{1}{T}, -\frac{\mu v}{T}\right)$ which are the conjugate variable. This is the Legendre transformation of the entropy S, i.e.

$$\Gamma r = \frac{G}{T} = -S + \frac{M}{T} N^a \frac{\mu_a}{T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.31)}$$

where G is the Gibbs free energy defined as:

$$G = M - TS - \mu_a N^a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.32)}$$

The sum of the entropy and its Legendre transformation is equal to the product of the extensive charges and the conjugate variable, namely

$$s + \Gamma = \beta_i Q^i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.33)}$$

The ruppeiner metric is physically meaningful in the equilibrium thermodynamic fluctuation theory, i.e., the thermodynamic Riemannian curvature, in short, thermodynamic curvature for any given thermodynamic state fells us about the underlying statistical structure, calculation of the Ruppeiner metrics as well as their Riemannian curvatures for different thermodynamic systems have been carried out as an example, the ideal gas at fixed volume has the Ruppeiner metric in the following form [40].

$$ds^2 = \left(\frac{Cv}{T^2}\right) dT^2 + \left(\frac{1}{T}\frac{V}{p^2 K_T} dp^2\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.34)}$$

Where Cv is the heat capacity at constant volume and K_T is the isothermal compressibility of the system. Notice that we are in the coordinates (T, p) where p (= N / V), is the density. To figure out whether this system is interacting or not, we can do so either by coordinate transformation (i.e., rewriting it in the form of Euclidean metric, which assures the flat space, i.e. the zero curvature) or computing the scalar curvature of the metric in Eq (3.34). It is not always possible to do the former method if the given metric is very complicated, the formula for the latter is given. It turns out for the ideal gas that it is a non-interacting system which we of course know from our existing knowledge, curvature scalar for the ideal gas with the particle number rather than the volume as the fixed scale is also zero as found by Nulton and Salamon [41].

3.3.2 Weinhold metrics

Weinhold [42] was the first to introduce the geometrical concept into thermodynamics. In his approach the Riemannian metric is considered in the energy representation where extensive parameters of the subsystem of the subsystem is given by

$$N^i = (S, N^a) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.35)}$$

The Weinhold metric w_{ij} is defined as the second derivatives of the internal energy (in this case, mass M) with respect to the entropy and N^i as given in Eq. [3.35] explicitly.

The contravariant form of the Weinhold metric is given by

$$W^{ij} = \frac{\partial^2 M}{\partial N^i \partial N^j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.36)}$$

will not be positive definite due to the fact that the ordinary black holes have negative heat capacities [43]. The negative heat capacity is a property of the isolated self-gravitating systems such as stars and astronomical systems [44]. Stars and black holes display the same phenomenon in this aspect, their temperatures increase as they lose energy, which is in accordance with the virial theorem.

Remarkably, the Wieinhold and the Ruppeiner metrics are conformally related via temperature T as the conformal factor, mathematically

$$W_{ij} dN^i dN^j = T S_{ij} dQ^i dQ^i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.39)}$$

The conformal relation will be of great importance, when the Ruppeiner metric seems to be very difficult to handle, by this we mean that instead of computing the Ruppeiner metric, we will work out the Weinhold metric and turn it into the Ruppenier metric via T, which is defined as $T = \frac{\partial M}{\partial S}$. It is worth nothing that the Ruppeiner metricobtained by using the conformal relation will be in the Weinhold coordinates, but its geometry is the Ruppeiner geometry and turn it into the Ruppenier metric via T, which is defined as $T = \frac{\partial M}{\partial S}$. It is worth nothing that the Ruppeiner metric obtained by using the conformal relation will be in the Weinhold coordinates, but its geometry s the Ruppeiner geometry.

3.4 Riemannian geometry of critical phenomena

Thermodynamic systems can be mapped into appropriate Riemannian geometries. For systems such as the ideal gas, ferromagnetic one-d Ising model and the van der Waals gas, it is found that the curvature of this manifold is related to the correlation volume of the system. The correlation volume is conventionally calculated from a statistical "microscopic" description of systems. For other system such as the antiferromagnetic model and Kerr Newman black holes, the correlation volume is given a new interpretation. In the case of Kerr Newman black holes, it is interpreted as the average number of correlated Plank areas at the holes surface. This idea may be formally extended to understand phase transitions in black holes thermodynamics where there is not a completely developed microscopic theory "quantum gravity".

Black holes are objects of interest both theoretically and experimentally. They represent an extreme system where the gravitational force reigns supreme. Black holes are a prediction of general relativity and it is widely believed that there is one at the center of every galaxy. On the astronomical scale, black holes are believed to play important roles in the large-scale structure of space and time, playing a major role in galaxy formation. Theoretically black holes are of huge interest because close to their centers, the curvature of space time and energy density of matter are infinite according to Einstein's equations of classical gravity. It is expected that the laws of quantum gravity should hold under these conditions, therefore an understanding of black hole interactions is crucial to a unified view of the two pillars of modern physics (quantum physics and GR). In this respect, there is still quite some work to be done both theoretically and experimentally. Also, in the modern context of ADS/CFT black hole models are crucial to the understanding of the holographic principle and derivative matrix of the entropy. Up to second order in fluctuations of thermodynamics variables from equilibriums, the Ruppiner metric physically measures the likelihood of fluctuations from equilibrium. Based on calculations done on the classical ideal gas model, an equation similar to that of general relativity is postulated to relate the curvature of the thermodynamics geometry to the interaction strength of the physical system. For the 1-d antiferromagnetic Ising model, van- der Waals gas, this postulate further suggests that the interaction strength is proportional to the correlation volume of the system which is conventionally calculated in statistical mechanical treatments as giving the range of the correlation function G (r) for systems such as the 1-d ferromagnetic model a new but useful interpretation of correlation length has to be adopted. This new interpretation is extended to the Ruppiener geometry for Kerr Newman black holes.

Other approaches to thermodynamics geometries are also possible. In particular by appealing to invariance of thermodynamics under a change of thermodynamic potentials, a Legendre invariant geometry for the study new interpretation is extended to the Ruppiener geometry for kerr- Newman black holes.

Other approach to thermodynamics geometries is also possible. In particular by appealing to invariance of thermodynamics under a change of thermodynamic potentials, a Legendre invariant geometry for the study of black holes physics reveals the points at which the heat capacity at constant charge and angular momentum diverges. According to a classification of black hole phase transitions due to Davies, [49] these are also the phase transition points of black holes. This classification is not generally agreed upon and other classifications based on for instance a change in topology have been devised. These are not discussed in here, rather attention is devoted to the formalism of the geometric formulation of thermodynamics.

The main feature of an ideal gas is the absence of interaction therefore the result that the idea gas thermodynamic geometry has zero curvature suggest an association of curvation with interaction strength. In [40] relation is postulated as given.

3.4.1 Geometric Formulation of Fluctuation Theory

The goal of a geometric formulations of thermodynamics is to provide a representation of systems which is intrinsic and does not depend on any particular choice of variables. In the case of fluctuation theory, the main foundation of a geometric formulation is that the equilibrium states of a thermodynamic system can be represented by points on an n-manifold. Where n is the number of independent physical quantities required to fully specify the states of the system, for example n=2 for an ideal gas. This manifold is assumed differentiable except at phase transitions and critical points [45].

The second foundation builds on an understanding that the equilibrium value of the thermodynamic quantities fluctuates about a mean value a result which follows from statistical mechanics [51]. According to this theory given an open system A, with fixed volume V immersed in a large energy reservoir, the values of thermodynamic quantities x of Av will fluctuate around a most probable value y with a probability distribution W (x, y) $dy = Ce^{S(x,y)/KB}$, where S (x, y) is the total entropy. K^B is Boltzmann's constant and B is normalization constant. This is really just a consequence of the way entropy is usually defined. As shown in for a pure fluid the W (x, y) can be expanded around its maximum x up to second order to give a second moment:

$$W(x, y)\,dxdy = 2\pi \exp\left(-\frac{1}{2} g_{ij}\Delta y_i \Delta y_i\right) \ldots\ldots\ldots\ldots (3.40)$$

Where $g_{ij}(x) = -\frac{\partial^2 S}{\partial y_i \partial y_i}$. Since the entropy is maximized at x, $g_{ij}(x)$ is positive definite and is thus a natural candidate for a metric on the 2-manifold of a fluid system with line element:

$$ds^2 = g_{ij}\Delta_{yi}\Delta_{yi}) \ldots\ldots\ldots\ldots (3.41)$$

The physical interpretation of this line element is that likely configurations are closer to the mean configuration while unlikely ones are further apart. It is well known in Riemannian and Pseudo-Riemmanian geometry that a metric tensor leads to the manifold and it is natural to enquire what the curvature of a manifold. This fact plays a very important role in general relativity [47]. The curvature is an intrinsic property of the manifold and it is natural to enquire what the curvature of the fluctuating metric would mean for thermodynamic systems. A guide to a general answer to this question is given in [45] where theatric in temperature density (T, p) coordinates for a classical ideal gas given by:

$$ds^2 = \frac{Cv}{kBT^2} dT^2 + \frac{V}{kBT_P^2 K_T} dP^2 \ldots\ldots\ldots\ldots (3.42)$$

Where C_v and K_T are the heat capacity at constant volume and isothermal compressibility respectively. The equation states is $P = pk_BT$ and $C_V = Nf(T)$. By making a change of coordinates $y_1 = (2V_P)^{\frac{1}{2}}(\cos\frac{T}{2} + \sin\frac{\tau}{2})$ and $y_2 = (2Vp)^{\frac{1}{2}}(\cos\frac{\tau}{2} - \sin\frac{\tau}{2})$ where $\tau = \int_C^T \frac{\int(T)}{K_BT^2}\frac{1}{2}dT$ and C is an arbitrary constant positive temperature, one gets that the line element for an ideal gas reduces to $ds^2 = dy_1^2 + dy_2^2$ which is just the line element of 2-d euclidean space with zero curvature.

The main feature of an ideal gas is the absence of interactions therefore the result that the ideal gas thermodynamic geometry has zero curvature suggests an association of curvature with interaction strength. In [45] relation is postulated as given by:

$$VK(x) = q1(x) \ldots\ldots\ldots\ldots (3.43)$$

This is the fundamental equation of this geometry and q is a constant that only can be determined from experiment.

$$K(x) = \frac{-1}{2g^{\frac{1}{2}}}\left[\frac{\partial}{\partial T}\left(g^{\frac{-1}{2}}\frac{\partial g_{pp}}{\partial T}\right) + \frac{\partial}{\partial p}\left(g^{\frac{-1}{2}}\frac{\partial g_{TT}}{\partial P}\right)\right] \ldots\ldots (3.44)$$

is the Gaussian curvature, $g_{TT} = \frac{C_y}{K_BT^2}$, $g_{pp} = \frac{V}{K_BTP^2K_T}$, $g = g_{TT}g_{pp}$ *and* I *(x)* is an object that characterizes interaction strength. Using the scaling relationships for a fluid along a path of critical density pc, $\frac{Cv}{V} = At^{-a}$ and $K_T = Bt^{-\gamma}$, where A and B are constant and t is the reduced temperature, one using the experimentally calculated α=0.1 and γ= 1.19. [45].

$$\frac{-V}{2g^{\frac{1}{2}}}\frac{\partial}{\partial T}\left(\frac{-1}{g2}\frac{\partial g_{pp}}{\partial T}\right) = 0.21\,\frac{k\beta}{A}t^{a-2} \ldots\ldots\ldots\ldots (3.45)$$

A more difficult calculation, [45] yields a term that is negligible and from equation (3.43) it can be concluded that I $(p_c, t) = 0.21\frac{k_b}{qA}t^{a-2}$. Therefore I (X) has the same dimensions and critical exponent as

ξ 3 where ξ is the correlation length. This suggests an identification of I(X) with ξ^d for any physical system. We note that this is merely an hypothesis whose validity can only be tested with experimentally determined values of correlation volumes for well known system. In the next section a summary of this hypothesis as applied to the 1−d ising model is given.

3.4.2 Results For 1-D Ising Model

By the same reasoning applied above, the metric for the 1−d ising model is given by $ds^2 = \frac{1}{T}\frac{\partial S}{\partial T}dt^2 + \frac{1}{T}\frac{\partial H}{\partial M}dM^2$, where H is the external field and given by M is the magnetization. The Gaussain curvature K(x) is given an expression analogous to (3.45). To test the hypothesis that K (x) obtained from geometry is directly related to the correlation volume, a numerical calculation on the partition function for the 1-d ising Hamiltonian has been done [52]. For the ferromagnetic ising model with coupling constant J>0 it is found that the correlation length ξ_G obtained from geometry is in excellent agreement with the known value of ξ which give the range of spin- spin correlations G (r) never deviating by more than one lattice constant [52]. When J=0, the manifold of thermodynamic states is effectively one dimensional which implies a curvature of zero, agreeing with the expectation that curvature is proportional to interaction strength. For the antiferromagnetic case with J<0, the results are not quite accurate in comparison with the usual definition of the correlation length. Nevertheless in [52] it is argued that ξ G can be interpreted physically as given the average length due to interactions of clusters of aligned spins.

For systems such as the Vander-waals gas and ideal paramagnet there is a similar agreement between the curvature and the correlation length.

Taking these formulations are containing some useful physics, this geometric model of analyzing the thermodynamic of systems can lead to insight about phase transitions is black hole where very little is understood about the microscopic processes that govern the interactions in black holes. In the next section, a quick review of black hole thermodynamics is given.

3.4.3 Review Of Black Hole Thermodynamics

According to Einstein's theory of General relativity the gravitational field manifests as curvature of space and time due to energy and momentum. In [48] it demonstrated that an analysis of Einstein field equation combined with the thermodynamics of stars reveal that the final start of collapse of collapse of a star about eight time more massive than our sun results in objects known as black holes, when a spherically symmetric collapsing star of mass M shrinks beyond a radius of $\frac{2GM}{c^2}$ the gravitational field becomes too strong that even light cannot escape this radius, and a black hole is inevitably forms. This is known as a Schwarzschild black hole and is uniquely specified by its mass. In this case the radius $\frac{2GM}{c^2}$ is known as the event horion. In the more general case after the collapse of a star of the black hole formed is uniquely determined by its mass M, angular momentum L and charge Q. This is known as a kerr-Newtonian black hole. Another black hole of interest is the BTZ black hole which is the black hole solution to Einstein equation in 2+1 dimensions with negative cosmological constant. However, this is also specified by its mass, charge and angular momentum. This state of affairs i.e. characterization by three parameters (M, L, Q) leads to a thermodynamic representation of black holes.

Since the final state of a black hole is uniquely determined by three parameters regardless of the structure of star that resulted in the hole, a given configuration of (M, L, Q) can be a result of several unicostate [49]. This idea leads to the notion of black hole entropy. The details are omitted here, but calculations from pure classical considerations give the entropy of a black hole as infinite a nonsensical result that was rescued by Steven Hawkings [46]. A true understanding of Hawking's result involves the complicated language of quantum field theory in curved space-time, but the resulting entropy is given in natural in units by

$$S=\frac{A}{4}=\pi(2M^2)-Q^2+2\sqrt{M^2-M^2Q^2-L^2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.46)$$

where A is the surface area of the event horizon. The second law of black hole thermodynamics is that in any process involving black hole the total surface area can never decrease [49]. A major issue in black hole thermodynamics is that of the definition of temperature, with details omitted here, the temperature of a black hole is defined to be $T=\frac{K}{2\pi}$,where $K=8\pi\frac{\partial M}{\partial A}$ is known as the surface gravity. This delimitation is essentially a zeroth law, and from equation (3.46) and the definitions given, the first law takes the form.

$$dM=TdS+\Omega dL+\Phi dQ\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.47)$$

Where

$$T=\frac{\partial M}{\partial S}\Omega=\frac{\partial M}{\partial L},\Phi\frac{\partial M}{\partial Q}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.48)$$

are the temperature, angular velocity and electric potential respectively. It must be emphasized that despite the fact that these thermodynamic notions are well defined, the internal structure of black holes is still poorly understood. In particular it is not still quite clear what is precisely meant by a phase transition in a black hole of course a full understanding can only be a well-developed theory of quantum gravity; Nevertheless, a lot of work has been done in this regard. From equations (3.48, 3.49) the heat capacity of a black hole at constant change and angular momentum, i.e. a measure of the energy transferred in a process between two equilibrium states that only changes the mass is given by:

$$C_{L,Q}=T\left(\frac{\partial S}{\partial T}\right)_{L,Q}=\frac{8MS^3T}{L^2+\frac{Q^2}{4}-8T^2S^3}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.49)$$

Davies [49] based his classification of phase transitions in black holes on this heat capacity. This is not generally agreed upon and this issue re-surfaces in the geometric formulation as well.

3.4.4 Fluctuation Geometry For Black Holes

As reviewed in the previous section a Kerr-Newman black hole is characterized by its mass M, angular momentum L and charge Q. Also, from the formula for the entropy given in (3.46) fluctuation geometry based on the entropy similar to that for the ideal gas black hole thermo-dynamics can be easily obtained. Before this is explored, a subtlety involved in the formalism must be addressed. We note that other thermo-dynamic geometries can be obtained based on the Hessian matrix of other thermodynamic potentials. A common one is Winhold geometry based on the internal energy [50]. It is a well-known fact from thermodynamics that the physical description of systems does not depend on the thermodynamic potential being used. It turns out that Ruppiener geometry is not Legendre invariant and this has led to a formulation of Legendre invariant geometry for black holes.

Here both results from the Ruppiener geometry and Legendre invariant geometry are summarized in [50]. After some differential geometric and physical consideration, it is obtained that the Legendre invariant metric on the phase space of thermodynamic variables for black holes is given by:

$$ds^2 = (MS_M+QS_Q+LS_L)\ (S_{MM}dM^2-S_{QQ}D^2-S_{LL}dL^2-2S_{QL}dQdL) \quad \text{.............. (3.50)}$$

where subscripts denote partial differentiation.

Using the entropy formula (3.46) and introducing coordinates(x^1,x^2,x^3) = (M, J, R), the line element for the Ruppiener geometry based on the hessian of the entropy is given by:

$$ds^2 = g\mu v dx^{\alpha} dx^{v} \quad \text{.. (3.51)}$$

where = $g\mu v = -\frac{8\pi}{L_r^2}\frac{\partial^2 S}{\partial x^{\mu}\partial xv^{-}}$ and L_P is the Planck length.

Next it is explored how the different formulations may yield insight into critical phenomena in black holes.

3.5 Critical Phenomena In Black Holes

3.5.1 Legendre invariant geometry

Reissner-nordstorm black hole

This is a black hole with L = 0, i.e. no angular momentum. It is spherically symmetric with two horizons at $r\pm = M\pm\sqrt{M^2 - Q^2}$ From (3.46) the entropy is S = π(M $+\sqrt{M^2 - Q^2}$, the heat capacity (3.49) in terms of r$\pm$ is $C_Q = -\frac{2\pi^2 r^2+(r+-r-)}{r1-3r}$ the scalar curvature in this case for the metric (3.50) is given by:

$$R= \frac{(r_+^2-3r-r_++6r_-^2)(r_+-r_-)^2}{\pi^2 r_+^3\left(r_+^2+3r_-^2\right)^2(r_+-3r_-)^2} \quad \text{.. (3.52)}$$

in the extremal limit which is a zero-temperature black hole, the scalar curvature is zero. From the expression [3.52] it is deduced that the scalar curvature diverges at $r_+ = 3r_-$ which is precisely where the heat capacity diverges signaling a second-order phase transition.

3.5.2 Kerr Black Hole

This is a neutral black hole corresponding to Q = 0. It represents a stationary, axially symmetric, rotating black hole with two horizons located at r$\pm = M \pm \sqrt{M^2 - L^2/M^2}$, the entropy [3.46] is S = $2\pi(M^2 + \sqrt{M^4 - L^2})$, the heat capacity (3.49) in terms of r$\pm$ is given by $C_L \frac{2\pi^2 r + \left(r_+^2+r_-\right)^2 (r_+-r_-)^3}{\left(r_+^2-6r_+r_--3r_-\right)^2}$. The scalar curvature obtained from the legendre invariant metric is given by:

$$R=\frac{(3r_+^3+3r_+^2+r_-+17r+r^2+9r_-^3)(r_+-r_-)^3}{2\pi^2 r+(r_++r_-)^4(r_+^2-6r_+r_--3r_-^2)^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3.53)$$

In this case the scalar curvature is found to diverge when $r_+^2-6r_+r_--3r_-^2$=0 which is exactly where the heat capacity diverges signaling a second-order phase transition.

3.5.3 General Kerr-Newman Black Hole

This represents the most general rotating and charged black hole with horizon at $r\pm = M\pm\sqrt{M^2-\frac{L^2}{M^2}-Q^2}$. The scalar curvature obtained in this case is R$\alpha\frac{1}{D}$ where

$$D\alpha A(M,Q,L)\left[2M^6-3M^4Q^2-6M^2L^2+Q^2L^2+2(M^4-M^2Q^2-L^2)^{\frac{3}{2}}\right]^2\ldots\ldots\ (3.54)$$

and A (M, Q, L) is always positive when $M^4 \geq M^2Q^2+J^2$ as required by the cosmic censorship hypothesis (which is a limit proposed by Roger penrose to avoid the presence of naked singularities in our universe [49]. The term in the squared brackets is the denominator of the heat capacity, therefore one concludes that the heat capacity diverges when the curvaure scalar diverges, also signaling a second order phase transition.

3.6 Ruppiener geometry

As explained above, concerns have been posed about the Legendre invariance of the Ruppiener metric nevertheless as discussed in the first few sections there are hints that there is some physics to be learnt from the Ruppiener metric, it is explained in [47] now a different interpretation of the curvature in terms of the number of correlated Planck lengths at the surface of the black hole is possible. Since the issue of phase transitions in black holes is not really well understood, this contrasting point of view is summarized here in terms of the entropy the first law (8) implies $\frac{1}{T}=\left(\frac{\partial S}{\partial S}\right)L,Q,-\frac{\Omega}{T}=\left(\frac{\partial S}{\partial T}\right)M,Q,-\frac{\Phi}{T}=\left(\frac{\partial S}{\partial L}\right)L,M$. Defining (α,β)=$\left(\frac{L^2}{M^4},\frac{Q^2}{M^2}\right)$ and (K, P) = $(\sqrt{1-\alpha-\beta},\sqrt{1+\alpha})$. The temperature is easily shown to be T= $\frac{4K}{(K^2+2K+P^2)M}$ and the cosmic censorship hypothesis imply $\alpha+\beta<1$ also for later use, we define:

$A=-2K^3-3K^2-2P^2K+2K-3P^2+4$ and $B=K^3+P^2K-K+1$

Since the Ruppiener geometry is based on fluctuations away from equilibrium, several cases as analyzed in [47] are now summarized.

3.6.1 M, L and Q fluctuating

When M, L and Q, fluctuates as demonstrated in [49], the black hole is unstable and it is not appropriate to use a fluctuation theory based on second order fluctuating moments here.

3.6.2 M Fixed, L, Q Fluctuating

In this case the metric corresponds to a 2-d geometry and the curvature is given in terms of the variables defined above by:

$$R = \frac{K^5 + P^2K^3 - 2K^3 - 2K^2 + 3P^2K+2}{4\pi KB^2} x \left(\frac{M_P}{M}\right)^2 \ldots\ldots\ldots\ldots\ldots (3.55)$$

where M_P, is the Planck mass. It turns out that this curvature never diverges in the physical regime allow by cosmic censorship [47].

3.6.3 L fixed, M and Q fluctuating

The metric for this case is also a 2-d geometry with scalar curvature:

$$\mathrm{R} = \frac{1}{2\pi K A^2} g(K,P) x \left(\frac{M_P}{M}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.57)$$

where g (K, P) is a polynomial function in its arguments and hence has no irregular behaviour. The curvature diverges only in the extremal limit K=0. By incorporating fluctuations into thermodynamics, a fluctuation geometry which can lead to insight into critical phenomena in black holes can be developed. The curvature scalar associated with a geometry based on the second derivative of the Hessian matrix of the entropy found for systems such as the ferromagnetic 1-d Ising model and the classical ideal gas is found to be proportional to the correlation volume. The correlation length is in accord with that usually defined as the typical length scale of spatial correlations G(r) derived from statistical mechanical models. In the case of the antiferromagnetic 1-d Ising model, this is not quite the case and a new interpretation of the correlation length as the average length of correlated spins has to be adopted. The is particularly relevant in the application to Kerr-Newman black holes where the correlation area obtained for the Ruppeiner Geometry is interpreted as the average number of correlated Planck area at the surface of a black hole. This interpretation is of course speculative and can only be confirmed by a microscopic theory/ experimental data hole. The physical interpretation of the Ruppeiner metric is that it measures at the likelihood of fluctuations from equilibrium metric is that it measures the likelihood of fluctuation from equilibrium to second order. It is not quite clear how higher order fluctuations could be incorporated into the theory. Also there seems to be no real physical motivation for this formulation of thermodynamics, only that it seems to second order. It is not quite clear how higher order fluctuations could be incorporated into the theory. Also there seems to be no real physical motivation for this formulation of thermodynamics, only that it seems to predict a way of understanding correlation length and predicting criticality. In particular it is not well understood how this method is related to the more standard renormalization group technique for studying critical behaviour.

By invoking Legendre invariance i.e., the well-known fact that thermodynamics does not depend on the choice of potential chose to study a system, Legendre invariant geometries similar to the Ruppiener geometries can be developed. For Kerr-Newman black holes, the scalar curvature of this geometry diverges at point where the heat capacity at constants angular momentum and charges diverges. According to the Davies classification of phase transitions in Black diverges. According to the Davies classification of phase of phase transitions in Black holes, this implies that the scalar curvature diverges at continuous phase transition points of Kerr-Newman black holes. Some authors disagree with the Davies classification and this issue may not be settled until when black holes are better understood. The importance of critical phenomena is that it seems to indicate that there is some physics that can be learnt from this formulation. It has been shown that the scaling behaviour of the scalar curvature and heat capacities in the extremal limit of Kerr-Newman black hole is equivalent to that for a 2-d fermi gas

[47]. Although no microscopic connection has been made, future work in this area should be directed at understanding how these correspondences come about. This may lead to insight on a correct microscopic theory of black holes.

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CHAPTER 4
Different families of black holes

Having introduced the Ruppeiner and the Winhold metrics in Chapter 3 we can now compute them for 4 different families of black holes. In order to do so, we need an expression for the entropy (for the Ruppeiner metrics) and the mass (for the Weinhold metrics). We will also use the mass formula to calculate the intensive parameters such as temperature T, angular velocity **Ω** and the electric charge **Φ** as well as the heat capacity C for each black hole. Some thermodynamic curvature scalars obtained from the Ruppeiner metrics will be plotted and discussed.

Smarr [53] in 1973 found the mass formula which contains all the information about the thermodynamic state of the Kerr-Newman black holes as

$$M = \sqrt{\frac{1}{4}\frac{A}{4\pi} + \frac{4\pi}{A}\left(J^2 \ + \frac{Q^4}{4}\right) + \frac{Q^2}{4}} \text{……………………} (4.1)$$

Where J and Q are the black hole's spin and the electric charge respectively is the surface (event horizon) area of the black hole which is expressible in terms of the hole's entropy as

$$S=\frac{1}{4}K_B A \text{…………………………………………………} (4.2)$$

by inserting $A = \frac{4S}{K_B}$ in Eq. (4.1) with K_B=1/π; it is found that the algebra becomes nicely simplified-this simplicity turns out to be extremely useful when we proceed to compute the Ruppeiner and the Weinhold metrics of these black holes as it reduces the degree of complication and tediousness. The simplified Smarr's mass formula takes the form:

$$M = \sqrt{\frac{S}{4} + \frac{1}{S}\left(J^2 \ + \ \frac{Q^2}{4}\right) + \frac{Q^2}{2}} \text{………………………} (4.3)$$

We will use this relation for obtaining certain thermodynamic properties of the Reissner-Nordstrom, the Kerr and the Kerr-Newman black holes. We will start with a theoretical BTZ black hole.

HEAT CAPACITY OF BTZ BLACK HOLE.

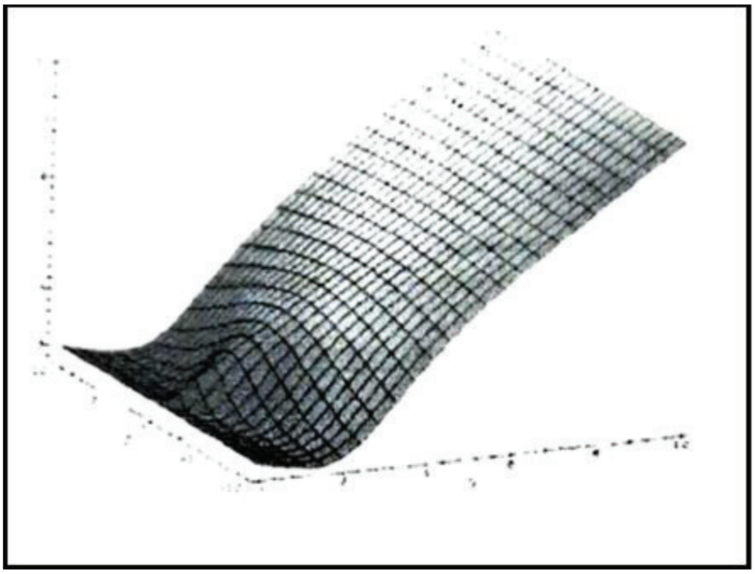


Figure 4.1:

Figure 4.1: Heat capacity of the BTZ black hole as a function of S and J. The C (S, J) is truly positive function for any values of S and J. It is notable that at the extremal limit $J=2S^2$, which is the same as J=M the temperature vanishes. Hence there is no heat capacity.

4.1: BTZ black hole

For the sake of simplicity, we will use $l= 1$ (that is we use $l = 1$) in Eq (2.20). Therefore, we have the event horizons of the BTZ black hole in the form:

$$r\pm= \sqrt{\frac{M}{2}(1 \pm \Delta)} \qquad (4.2)$$

where

$$\Delta= \sqrt{1-\left(\frac{J}{M}\right)^2} \qquad (4.4)$$

The entropy of the BTZ black hole (note that we use $K_B = 2 / \pi$ for this particular case) is

$$S= \frac{A}{2\pi} = \frac{L}{2\pi} = \frac{2\pi r_+}{2\pi} = \sqrt{\frac{M}{2}(1+\Delta)} \qquad (4.6)$$

where L is the length of the horizon r_+. By solving Eq. (4.6) for M we have the mass in terms of the entropy for the BTZ black hole as

$$M= S^2 + \frac{J^2}{4S^2} \qquad (4.7)$$

The temperature of the BTZ black hole is given by

$$T=\frac{\partial M}{\partial S} = 2S - \frac{J^2}{2S^2} \qquad (4.8)$$

and its angular velocity is

$$\Omega = \frac{\partial M}{\partial J} = \frac{J}{2S^2} \quad (4.9)$$

Its heat capacity takes the form:

$$C = \frac{\partial M}{\partial T} = T\frac{\partial S}{\partial T} = \frac{T}{\frac{\partial T}{\partial S}} = \frac{T}{\frac{\partial^2 M}{\partial S^2}} \quad (4.10)$$

$$\Rightarrow C = \frac{S(4S^4 - J^2)}{4S^4 + 3J^2}$$

It is readily seen that the heat capacity of the BTZ hole is positive. We have found that for this black hole it is more convenient to evaluate the Weinhold metric and use the conformal relation to obtain the Ruppeiner metric. The Weinhold line element reads

$$ds_w^2 = \left(2 + \frac{3J^2}{2S^4}\right) dS2 - \left(\frac{2J}{S^2}\right) dSdJ + \left(\frac{1}{2S^2}\right) dJ^2 \quad (4.11)$$

The Weinhold metric turns out to be a non-flat metric but we are not interested in its curvature scalar, so we proceed to calculate the Ruppeiner metric for the BTZ black hole by using $ds^2{}_R = \frac{1}{T} ds^2$ which takes the form:

$$ds_R^2 = \left(\frac{(4S^4+3J^2)}{(4S^4-J^2)S}\right) dS^2 - \left(\frac{2J}{4S^4-J^2}\right) dSdJ + \left(\frac{S}{4S^4-J^2}\right) dJ^2 \quad (4.12)$$

and it is diagonalizable as

$$ds_R^2 = \left(\frac{1}{S}\right) dS^2 + \left(\frac{S}{1-u^2}\right) du^2 \quad (4.13)$$

with

$$u = \frac{J}{2S^2} - \leq u \leq 1 \quad (4.14)$$

This metric yields a flat Riemannian curvature, i.e., the space of thermodynamic state is a flat space.

4.2 Reissner-Nordstrom black hole

From the mass formula in Eq. (4.3), the Reissner-Nordstrom black hole has

$$M = \sqrt{\frac{S}{4} + \frac{1}{S}\frac{Q^2}{4} + \frac{Q^2}{2}} \quad (4.15)$$

fortunately, it can be simplified as

$$M = \frac{\sqrt{S}}{2}\left(1 + \frac{Q^2}{S}\right) \quad (4.16)$$

The temperature and the heat capacity for the Reissner-Nordstrom black hole are as follow:

$$T=\frac{\partial M}{\partial S}=\frac{1}{4\sqrt{S}}\left(1-\frac{Q^2}{S}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.17)$$

And

$$c=-2S\left(\frac{1-\frac{Q^2}{S}}{1+\frac{3Q^2}{S}}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.18)$$

RUPPEINER METRIC FOR THE RN BLACK HOLE

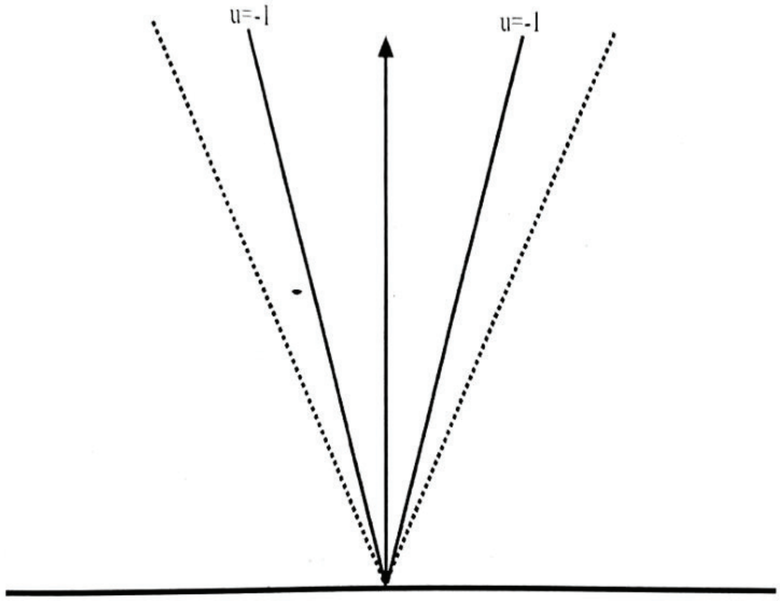


Figure 4.2:

Figure 4.2: The Ruppeiner metric for the RN black hole as seen in Minskowski space when Rindler coordinates are used. The shaded area represents the Lorentzian metric, i.e., the metric of $(-+)$ signature.

and the electric potential takes a simple form:

$$\Phi=\frac{\partial M}{\partial Q}=\frac{Q}{\sqrt{S}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.19)$$

It is obvious that the heat capacity for the Reissner-Nordstrom hole is a negative quantity. Notice that there is an absolute-zero temperature associated with the hole when Q=$\sqrt{S}$, which is the extremal limit. We next calculate the Weinhold metric and otain

$$ds\frac{2}{W}=\frac{1}{8S^{\frac{3}{2}}}\left[-\left(1-\frac{3Q^2}{S}\right)dS^2-8QdSdQ+8SdQ^2\right]\ldots\ldots\ldots\ldots(4.20)$$

We can diagonalize this metric by using

$$\xi=\frac{Q}{\sqrt{S}},-1\leq\xi\leq 1\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.21)$$

which yields

$$ds_W^2 = \frac{1}{8S^{\frac{3}{2}}}[-(1-\xi^2)dS^2 + 8S^2 d\xi^2] \ldots\ldots\ldots\ldots\ldots (4.22)$$

We can turn the Weinhold metric above into the Ruppeiner metric in the same coordinates via the conformal relation as

$$ds_R^2 = \left(\frac{-1}{2S}\right) dS^2 + \left(\frac{4S}{1-\xi^2}\right) d\xi^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.23)$$

which has the same properties as the Ruppeiner metric obtained directly from the entropy representation where

$$S\,(M, Q) = 2M^2 - Q^2 + 2M^2\sqrt{1-\frac{Q^2}{M^2}} \ldots\ldots\ldots\ldots\ldots (4.24)$$

S (M, Q) is just an inversion of the mass formula in Eq. (4.16) By computing its curvature scalar, it is found that the Ruppeiner metric for the RN black hole is a

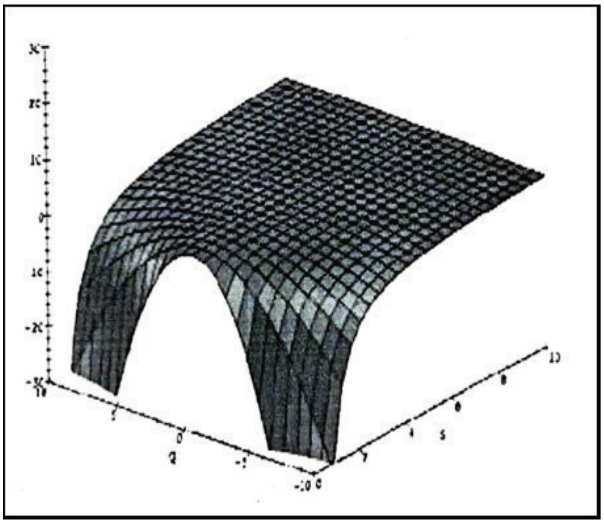


Fig. (A)

Figure 4.3: (A)

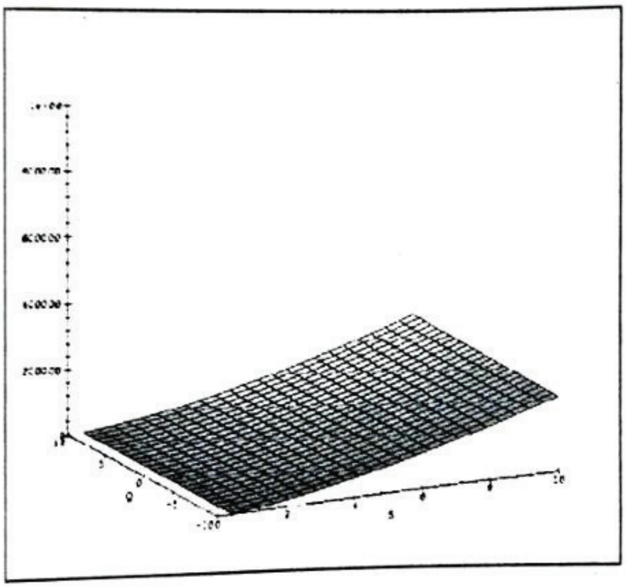


Fig. (B)

Figure 4.3: (B)

Figure 4.3: Heat capacity of the RNadS black hole as a function of entropy S and electric charge Q for 2 different values of *l*, i.e., we have set *l* = 1 in Fig. 4.3 (A) and *l* =0.01 in Fig. 4.3 (B). The heat capacity of the RNadS black hole vanishes at the extremal limit, whereas in the case of a small value of *l* (large A) as in Fig. 4.3 (B). The heat capacity becomes linear in the entropy.

flat metric, which is the same as that of the BTZ black hole. We now introduce new coordinates

$$\tau = \sqrt{2S} \text{ and } \sin = \frac{\sigma}{\sqrt{2}} = \xi \quad \text{.................................... (4.25)}$$

The line element in Eq. (4.23) then reads

$$ds^2 = -dr^2 + r^2 d\sigma^2 \quad \text{....................................... (4.26)}$$

This is a time like wedge in a Minkowski space as seen in Rindler coordinates, see Fig. (4.2).

For the sake of completeness, we consider also the Reissner-Nordstrom black hole with a negative cosmological constant (the Reissner-Nordstrom-anti-de Sitter or RNadS for short) and we use the mass formula [54]:

$$M^2 = \frac{A}{16\pi} + \frac{\pi}{4}(4J^2 + Q^4) + \frac{Q^2}{2} + \frac{J2}{t^2} + \frac{A}{8\pi l^2}\left(Q^2 + \frac{A}{4\pi} + \frac{A^2}{32\pi l^2}\right) \quad \text{.............. (4.27)}$$

which J = 0 and K_B =1/π (hence S = $\frac{\partial}{4\pi}$) it simply becomes

$$\mathrm{M} = \frac{\sqrt{S}}{2}\left(1 + \frac{S}{J^2} - \frac{Q^2}{S}\right) \quad \text{... (4.28)}$$

The temperature associated with the RNadS black hole is given by

$$\mathrm{M} = \frac{1}{4\sqrt{S}}\left(1 + \frac{3S}{j^2} - \frac{Q^2}{S}\right) \quad \text{................................... (4.29)}$$

which vanishes in the extremal limit, namely at

$$\frac{Q^2}{S} = 1 + \frac{3S}{l^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.30)$$

as well as the heat capacity for the RNadS black hole, which takes the from

$$C = 2S\left[\frac{S\left(\frac{3S}{l^2}+1\right)-Q^2}{S\left(\frac{3S}{l^2}-1\right)+3Q^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.31)$$

The heat capacity of the RNadS black hole become linear in S in the limit of $l \to \infty$. We display heat capacities of this black hole for 2 different values of 1 in Fig. (4.3) and its electric charge is given by

$$\boldsymbol{\Phi} = \frac{\partial M}{\partial Q} = \frac{Q}{\sqrt{S}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.32)$$

We are now in the position to compute the Ruppeiner metric for the RNadS black hole; we will do so via the conformal relation as it is obviously simpler, in this case, to calculate the Weinhold metric directly from the mass formula in Eq. (4.28), we thus have

$$ds\frac{2}{W} = \frac{1}{8S^{\frac{3}{2}}}\left[-\left(\frac{3S}{l2} - \frac{3Q^2}{S}\right) dS^2 - 8QdSdQ + 8SdQ^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.33)$$

this metric can be diagonalized using the same coordinates as in Eq. (4.25) which yields the diagonal metric in the form

$$ds^2 = \frac{1}{\left(1+\frac{3r^2}{2l^2}-\xi^2\right)}\left[-\left(1 - \frac{3r^2}{2l^2} - \xi 2\right) dr^2 + 2r^2 d\xi^2\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.34)$$

This metric has a non-trivial geometry, namely there is a change of the signature of the metric at

$$\xi^2 = \frac{Q^2}{S} = 1 - \frac{3S}{l^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.35)$$

This corresponds to the stability properties of the thermodynamic system which change for sufficiently large black hole [55]. We show this in the Fig. (4.4). From the Ruppeiner metric we obtain the Riemannian curvature scalar in the form

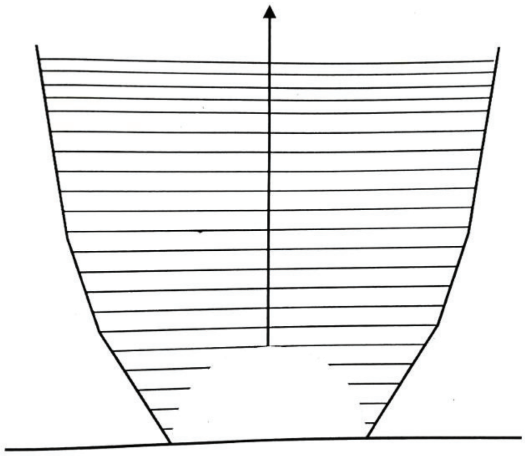
$$R = \frac{9}{12}\frac{\left(\frac{3S}{l^2}+\frac{Q^2}{S}\right)\left(l-\frac{S}{l^2}-\frac{Q^2}{S}\right)}{\left(1-\frac{3S}{l^2}-\frac{Q^2}{S}\right)\left(1-\frac{3S}{l^2}-\frac{Q^2}{S}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.36)$$

which has a divergence in the extremal limit and along the curve where the metric changes its signature. The 3D plot of this is in Fig. (4.5). This RNadS black hole comes the ordinary RN black hole in the limit of $l \to 0$.

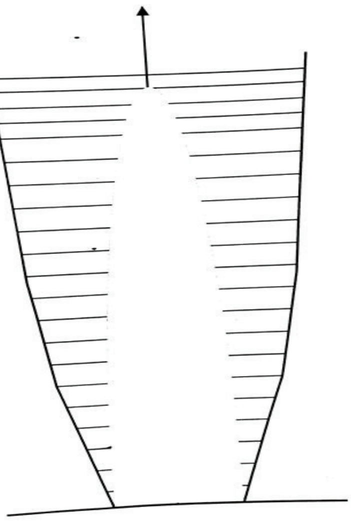
4.3 Kerr black hole

The mass formula for the Kerr black hole is given by setting Q=0 in Eq. (4.3) which reads.

$$M = \sqrt{\frac{S}{4} + \frac{J2}{S}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.37)$$



(Fig. A) 4.4A



(Fig. B) 4.4 B

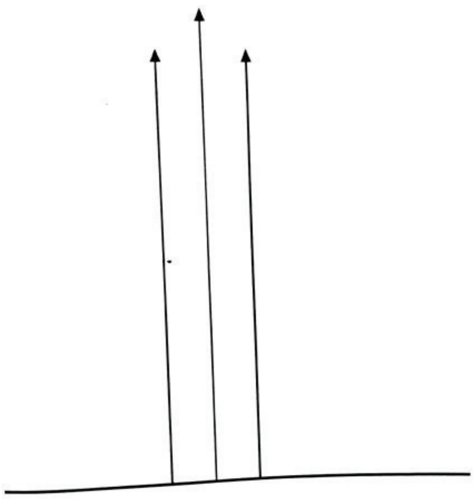
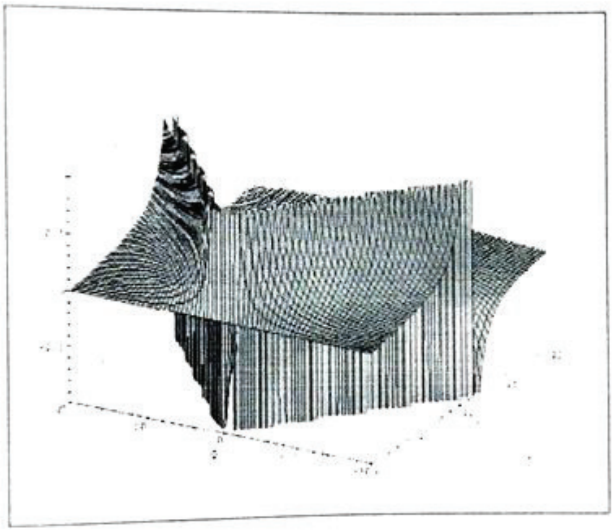
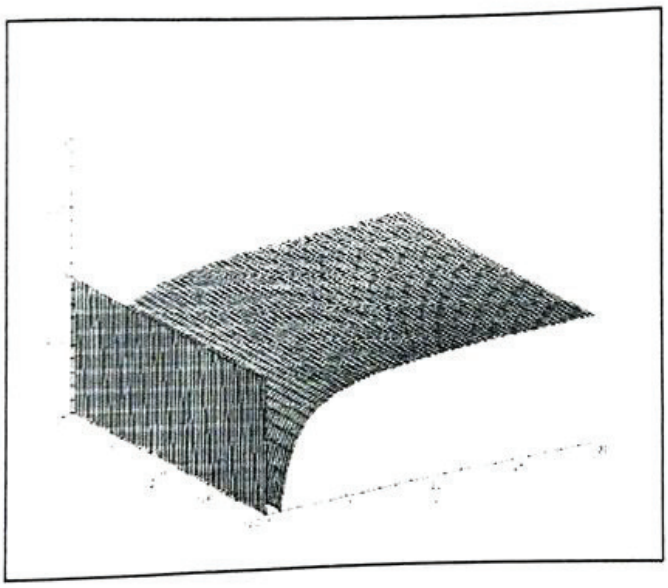


Fig. C) 4.5 C

Figure 4.4: The RNadS black hole's Ruppeiner geometry the shaded areas represent the Lorentzian metric while the strips are the Riemannian metric which is a positive definite metric with the (++) signature. Fig. (A) is when l=1 which is the when the cosmological constant $\Lambda = -1$, Fig. (B) represents the geometry when the value of 1 is large. In Fig.(C) we $l \rightarrow \infty$ which means that the cosmological constant is vanishing thus it reduces to the case of RN black hole where its Ruppeiner metric is a Lorentzian metric.



(Fig. A) 4.5



(Fig. B) 4.5

The temperature takes the form:

$$T = \frac{\partial M}{\partial S} = \frac{1-\frac{4j^2}{S^2}}{4\sqrt{S+\frac{4j^2}{S}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.38)$$

and it vanishes in the extremal limit, namely S = 2J or in terms of J and M it is J = M^2. The angular velocity of the hole is given by:

$$\Omega = \frac{\partial M}{\partial J} = \frac{2j}{S\sqrt{S+\frac{4J^2}{S}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.39)$$

On inversion of Eq. (4.37) we obtain the entropy in the form

$$S = 2M^2 + 2M^2\sqrt{1-\frac{J^2}{M^4}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.40)$$

Hence obtain the Ruppeiner metric in the following form

$$ds_R^2 = \frac{2}{\left(1-\frac{J^2}{M^4}\right)^{\frac{3}{2}}}\left\{-2\left[\left(1-\frac{J^2}{M^4}\right)^{\frac{3}{2}} + 1 - \frac{3J^2}{M^4}\right]dM^2 - \left(\frac{4J}{M^3}\right)dMdJ + \left(\frac{1}{M^2}\right)dJ^2\right\} \ldots\ldots (4.41)$$

by the virtue of the coordinate transformation, it is reduced to the diagonal form as

$$ds_R^2 = -2\left(1+\frac{2}{\sqrt{1-\pi^2}}\right)dM^2 + \left(\frac{2M^2}{(1-\pi^2)^{\frac{3}{2}}}\right)d\pi^2 \ldots\ldots\ldots (4.42)$$

where

$$\eta = \frac{J}{M^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.43)$$

It turns out that the Ruppeiner geometry of this black hole is curved-it scalar curvature is given by

$$R = \frac{1}{4M^2}\frac{\sqrt{1-\frac{J^2}{M^4}}-2}{\sqrt{1-\frac{J^2}{M^4}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.44)$$

The above curvature scalar has a divergence in the extremal limit of the Kerr black hole, namely at J=M^2 which is where the extremal limit of the black hole.

4.4 Kerr-Newman black hole

The Kerr-Newman black hole has the mass formula given by Eq. (4.3) namely

$$M\,(S, J, Q) = \sqrt{\frac{S}{4} + \frac{1}{S}(J^2) + \frac{Q^2}{4} + \frac{Q^2}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.45)$$

Thermodynamics of this black hole can be described by the variation in mass M as

$$dM = TdS + \Omega dl + \mathbf{\Phi} dQ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.46)$$

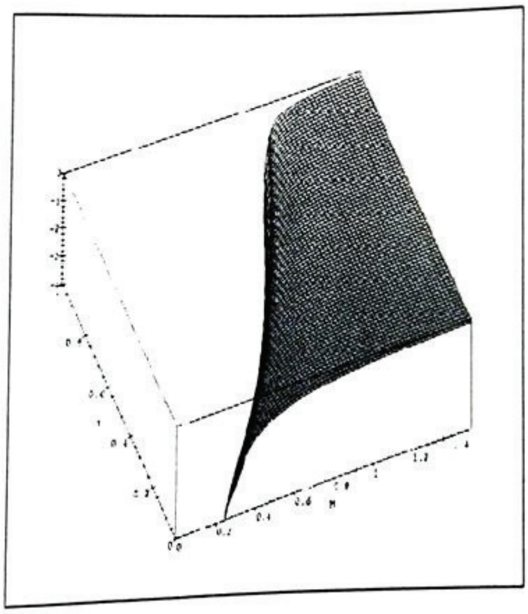


Figure 4.6

Figure 4.6: The curvature scalar for the Kerr black hole as a function of M and J. It is noticeable that a divergence of the curvature occurs along J=M^2 which is an extremal limit of this black hole.

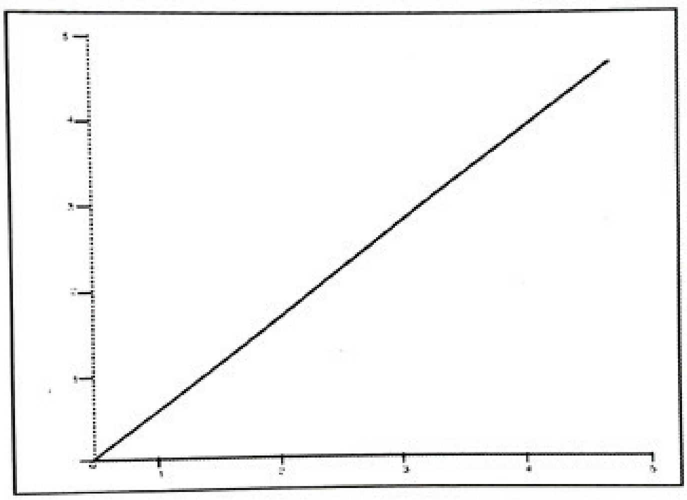


Figure 4.7

Figure 4.7: Heat capacity for the KN black hole for constant J and Q, we here set J = Q = 1.

which is indeed the second law of thermodynamics where the intensive parameters T, Ω and **Φ** are given by

$$T= 1\frac{1}{4}\frac{-S^2+4J^2+Q^4}{S\sqrt{S(4J^2+S^2+2SQ^2+Q^4)}} = \frac{1}{4}\frac{(S^2-4J^2+Q^4)}{2MS^2} \ldots\ldots\ldots\ldots\ldots \quad (4.47)$$

with the entropy S being just the inversion of Eq. (4.46) which takes the form:

$$S = 2M^2 - Q^2 + 2M^2\sqrt{1 - \frac{Q^2}{M^2} - \frac{J^4}{M^4}} \quad \text{.......................... (4.48)}$$

The other intensive parameters are its angular velocity **Ω**

$$\Omega = \frac{2J}{\sqrt{S(4J^2 + S^2 + 2SQ^2 + Q^4)}} = \frac{J}{MS} \quad \text{.................................. (4.49)}$$

It is readily seen that the larger the mass and entropy of the black hole, smaller the angular velocity, and as expected the angular velocity is proportional to the angular momentum of the black hole. The electric potential takes the following form:

$$\boldsymbol{\Phi} = \frac{Q(S + Q^2)}{\sqrt{S(4J^2 + S^2 + 2SQ^2 + Q^4)}} = \frac{Q(S + Q^2)}{2MS} \quad \text{............................. (4.50)}$$

This potential is reduced by mass and entropy of the black hole. As a matter of fact, the KN black hole has a fairly complicated form of heat capacity, therefore we will only show it graphically. The heat capacity is a function of S, J and Q-for simplicity we will consider it for constant angular momentum and charge. Thus, by setting J = Q = 1 we have found that the heat capacity becomes linear in entropy, see plot in Fig (4.7).

The Ruppeiner metric of the Kerr-Newman black hole is very complicated and too tedious to work out by hand, therefore we utilize a computer program to ease the calculation. We will not present it here since it does not give any significance to the rest of this work. However, we have found that the KN black hole has curved geometries of both the Ruppeiner and the Weindhold metrics, the Ruppeiner is found to be non-conformally flat [55].

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CHAPTER 5
Summary and Conclusion

This thesis is concerned with the knowledge to understand the various thermodynamical parameters in the study of black holes thermodynamics i.e. means of thermodynamics (Classical) fluctuation theory based on the concept of the Riemannian geometry. Contained in this volume are five chapters fallowed by a bibliography relevant to the subject of the thesis.

The first chapter is introduction. In this chapter the author has attempted to justify the reason for taking of the problem and has also mentioned the brief history of black holes, also introducing messages especially for those who might not be familiar with the subject of general relativity with Einstein's gravitational field. Black holes are probably the most mysterious objects in the general theory of Relativity that are still far from being fully understood. Whereas a quantum description of the nature of black hole space-time has remained elusive as of now apart from certain specific examples in loop quantum gravity (LQG), a lot of progress has never the less been made in the understanding of black hole thermodynamics. The Schwarzschild radius (Rs) has been attained; the object will have such a strong gravitational field that it will prevent even light from escaping out of its influence. Since photons cannot escape, no event occurring within the Schwarzschild radius can be communicated outside this radius.

The works of Penrose and Hawkins have shown that a still more efficient process of gravitational radiation can be achieved when two black holes comes in contact and merge together.

In the beginning of the second chapter, the author briefly presented the highlights of General Relativity with various thermodynamics parameters of black hole thermodynamics with uncertainly principle for black holes like BTZ, Kerr and Kerr-Newman (KN) black holes respectively. It is known that a black hole can be entirely described through the use of only three parameters the mass M, angular momentum J and charge Q. Though the mass of observed black holes ranges over several orders of magnitude, it is believed that a tends to be nonzero and significant, and that the Residual charge Q is nearly identically zero. Thus, we seek a metric for a spinning black hole with no charge. This is known as the Kerr metric, and within the equatorial plane in Boyer Lindquist coordinate. The BTZ discovered, in 1992, the black hole solution of Einstein's equation with a negative cosmological constant in 2+1 dimensions and without couplings to matter [29,20]. This discovery was rather surprising as there was no speculation that there would exist a black hole solution in 2+1 dimensions at that time, the BTZ black hole is known as a simple toy model for a number of studies including the spring and super gravity theory. It has been established that black hole entropy, as given by the celebrated Bekenstein Hawking area laws S=A/4 (in units of the Planck's area) acquires logarithmic correction due to the fluctuations in the black hole parameters, both at the quantum as well as classical level.

The third chapter is basically the main stream of this work, which is an application of the thermodynamic fluctuation theory to the black holes. [Results will be given in chapter four followed by conclusions.]. The main idea in this thesis is to exploit the Smarr [53] type relations between the black hole parameters to calculate entropy corrections via fluctuation theory. This can also take into account fluctuations of the angular momentum for rotating black holes which to the best of our knowledge have not been dealt with so for. A formula for such correction, involving the black hole response coefficients is also written down. This is illustrated for the example of BTZ black hole in chapter second. Thermodynamics and Geometry have been discussed with fluctuation geometry of black hole like Ruppiener, Riemannian geometry of critical phenomena with review of black hole thermodynamics.

The chapter four computes the results for 4 different families of black holes thermodynamics. The concepts of the BTZ, Reissner Nordstrom (RN), Kerr, Kerr-Newman (KN) black hole discussed. The Ruppeainer metric of the Kerr-Newman black hole is very complicated and too tedious to work out by hand. However, we have found that the KN black hole has curved geometrics of both the Ruppeiner and The Weindhold metrices the Ruppeiner is found to be non-conformably flat. The problem being too vast, the author has however tried of scratch only something for the black hole thermodynamics area.

In this thesis work, thermodynamic Riemannian geometries of various black hole families are studied, namely the Ruppeiner and Weinhold geometries. The Ruppeiner geometry is constructed on the basis of the fluctuation theory in equilibrium thermodynamics and it is conformably related to the Weinhold geometry via temperature in the mathematical expression. It is, however, only the Ruppeiner geometry that tells us the statistical interaction underlying the system under consideration. Although the black hole is believed to be a thermodynamic system, its statistical mechanical structure is lacking, partly due to own limited knowledge of gravity at the quantum level. The Ruppeiner geometry for the BTZ and RN families is flat (zero curvature), as expected because of the simpler structure they possess the thermodynamic scalar curvature diverges in the extremal limit in the Kerr, RNadS and KN families that possess a curved geometry. Most interestingly perhaps is the RNadS black hole family whose curvature is singular along the line where the stability properties change and it coincides with the RN family in the limit of vanishing cosmological constant. The Weinhold geometry gives rise to no physical meaning, it is curved for the BTZ, RN and RNadS families whereas the Kerr family has a flat. Weinhold geometry. For the sake of tidiness, the obtained results are tabulated in the fallowing page. We believe that these results are sensible and simpler than what one might have expected, although no statistical physics of black holes can be derived from these findings at this point in time. Finally, it is hoped that these results will find an interesting pattern in the future when the quantum theory of black holes is more concrete.

GEOMETRIC OF THE RUPPEINER AND WEINHOLDMETRICS BLACK HOLE FAMILY

Black hole family	**Ruppeiner metric**	**Weinhold metric**
BTZ	Flat	Curved
Reissner-Nordstrom	Flat	Curved
Reissner-Nordstrom-anti-de-Sitter	Curved	Curved
Kerr	Curved	Flat
Kerr-Newman	Curved	Curved

Table 5.1: Geometric of the Ruppeiner and Weinhold metrics for various black holes.

Bibliography

1. Hawking, S. W., Nature, 248, 30 (1974).
2. Hawking, S.W., Comm. Math, Phys. 43, 199 (1975).
3. Bekenstein, J.D., Phys. Rev. D7 2333 (1973).
4. Bardeen, J., Carter, B, and Hawking, S. W., Comm. Math. Phys.,31 161 (1973).
5. Hawking, S. W., Phys. Rev., D13 191 (1976).
6. Hawking, S.W., and Ellis, GF.R., The Large-Scale Structure of Space-Time (Cambridge University, Cambridge, London 1973).
7. Hawking, S.W., Nature (Londan) 248, 30 (1974).
8. Hawking, S. W., Commun. Math. Phys. 43, 199 (1975).
9. Bekenstein, J.D., Phys. Rev. D7, 947, 2333 (1973).
10. Penrose, R. and Floyd R. M., Nature 229,177 (1971).
11. Christodoulou, D., Phys. Rev. Lett. 25, 1596 (1971).
12. Hawking, S.W., Phys. Rev. Lett. 26, 1344 (1974).
13. Misner, C. W., Thorne, K.S. and Wheeler, J.A., Gravitation, W.H. Freeman and Company (New Yourk 1973).
14. Schwarzschild K. on the gravitational field of a mass point according to Einstein's theory Sitzungsber. Preuss. Akad. wiss. Phys.Math Kl., 1916. 189; http://www.geocities.com/theometric/schwarzschild.pdf. –
15. Droste J. The field of a single centre in Einstein's theory of gravi-tation, and the motion of a particle in that field. Ned. Acad. Wet.S.A.,1917 v. 19, 197;http://www.geocities.com/theormetria/Droste.pdf.
16. Hilbert D. Nachr. Ges. wiss. Gottingen, Math. Phys. Kl., v. 53, 1917/http://www.geocities.com/theometria/hilbert.pdf.
17. Weyl H. Ann. Phys. (Leipzig), 1917, v.54,117.
18. Brillouin M. The singular points of Einstein's universe. Journ. Phys Radium, 1923, v. 23, 43; http;//www.geocities.com theometria/brillouin.pdf.
19. Synge J.L. The gravitational field of a particle. Proc. Roy. Irish Acad., 1950, v. 53, 83.
20. Kruskal M.D. Maximal extension of Schwarzschild metric. Phys. Rev., 1960, v. 119,1743.
21. Szekeres G. On the singularities of a Riemannian manifold. Math. Debreca., 1960, v. 7, 285.

22. Mevittle G.C. Laplace's alleged "black hole" The Observatory, 1978, v.98,272: http://www.geocities.com/theometria/MeVittie.pdf
23. Crothers S.J. On the general solution to Einstein's vacuum field and its implications for relativistic degeneracy. Progress in Phys-ics, 2005, v.1, 68-73.
24. Crothers S.J. On the general solution to Einstein's vacuum field for the point-mass when 2-0 And its implication for relativistic cosmology. Progress in Physics, 2005, v.3, 7-18.
25. E.F. Taylor and J.A. Wheeler, Exploring Black Holes (Addison Wesley Longman, 2000).
26. R. Penrose, Rivista del Nuovo Cimento (1969).
27. S. Chandrasekhar, Mathematical Theory of Black Holes (Oxford University Press, 1983).
28. M. Bhat, S. Dhurandhar, and N. Dadhich, Journal of Astrophysics and Astronomy 6 (1985).
29. Banados, M. et.al., Phys. Rev. Lett. 69, 1849 (1992).
30. Banados, M. et. al., Phys. Rev. D48, 1506 (1993).
31. Bekenstein, J.D., Phys. Rev. D9 12 (1974).
32. Wald R. M., The Thermodynamics of Black Holes, Lecture Note, Enrico Fermi Institute and Department of Physics, University of Chicago.
33. Hehl, F. W. et. al., Black Holes: theory and Observation, Springer (Germany 1998).
34. Kiefer, C., gr-qe/011070.
35. Wald, R. M., http://www.livingreviews.org.Articles/volume4.2001-6wald., cited on 19 February 2003.
36. Greiner, W. et. al., thermodynamics and statistical Mechanics springer-verlag (New Your 1995).
37. Benenson W, et. al., Handbook of Physics, Springer (New York 2002).
38. Landan, L.D., Lifshitz, E.M., statistical Physics, Pergamon Press (Hungary 1980).
39. Ruppeiner, G, Physics. Rev. A20, 1608 (1979).
40. Ruppeiner, G., Rev. Mod. Physics. 67,605 (1995).
41. Nulton, J.D. and Salamon, P., Physics. Rev. A31,2520 (1985).
42. Weinhold, F., J. Chem Phys. 63, 2479 (1975).
43. Ferrara, S. et. al., hep-tp
44. Lynden- Bell, D, and Lynden- Bell, R. M., Mon. Not. R. astr.Soc. 181, 405-409 (1977).
45. G. Ruppeiner, Phys. Rev. A 20, 4 (1979).
46. S. W. Hawking, Phys Rev. D 13, 191 (1976).
47. G. Ruppeiner, Phys Rev. D 78, 024016 (2008).
48. C.W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation

49. P.C.W. Davies Rep. Prog. Phys, vol 41 (1978).
50. J.L. Alvarez, H. Quevedo, A. Sanchez, Phys Rev. D 77, 0804 (2008).
51. L.D. Landau and E.M. Lifschitz, Statistical Physics.
52. G. Rupeiner, Phy. Rev. Mod. A 24, 1(1981).
53. smarr, L., Phys. Rev. Lett. 30, 71 (1973)
54. Caldarelli, M.M., hep-th/9908022.
55. Aman, J., Bengtsson, I and Pidokrajt, N., Geometry of black hole thermodynamics, gr-qc/0304015 and General Relativity and Gravitation 35 (10) (2003) 1733.
56. *****

APPENDIX 1

Study of thermo-optical properties of black hole radiations

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Abstract: Black holes have become very important part of the solar system. Although most of the properties of the black holes have been predicted theoretically and they have been verified by observation. However, new observations are always found out, which require proper theoretical support. This article is a step to describe some of the thermos-optical properties of the black holes. The rotational spectra produced due to the gas which his sucked from the normal star and spirals towards the black hole's event horizon has also been investigated. The elementary black holes produced during Hadron collision and their properties have also been studied in this article.

Key words: Black holes, thermos-optical properties, elementary black holes.

1. Introduction: The black hole is a prominent member of the solar family found at the center of almost all the milky ways. It is difficult to say whether it is the cause of the birth of a solar system or it is one of the ultimate ends of the systems. Anything that crosses the boundary known as the event horizon becomes trapped due into it due to its enormous gravitational pull. There are various dimensions of explaining the concept of the black hole. One of the simple concepts is based on the principle of escape velocity. According to it if we consider a spherical body of mass M and radius R, then gravitational potential at the surface of the body is defined as: $V = -\frac{GM}{R}$ .Therefore the potential energy of a body of point mass m at the surface of the spherical body will be: = V m = $\frac{-GMm}{R} joul$. Thus amount of energy required to separate the point mass m from M by infinity distance will be given by solving the equation:

$$\frac{1}{2}mv - \frac{GMm}{R} = 0 \text{ ---(1)}$$

Where v is the velocity of the point mass m so that, it can go up to infinity distance from the surface of the spherical body. So, this equation gives the value of escape velocity $v=\sqrt{\frac{2GM}{R}}$. This velocity is called the escape velocity which depends on the mass and the size of the spherical body. If we consider the earth as the body of mass M and its radius R, then the escape velocity of the earth is:

$$V_{escape} = \sqrt{\frac{2GM}{R}} = 11200 \text{ meter /second ---(2)}$$

Since according to Einstein, the velocity limit of a particle is: C = velocity of light = 3×10^{8} m/s.

So, if we propose a body which could never allow any particle to escape from its surface, not even the light, then the value of escape velocity on the surface of this body would be equal to the velocity of light C where:

C^2=2GM/R--(3)

This gives $\frac{M}{R} \geq \frac{C^2}{2G}$=$6.75 \times 10^{26}$---(4)

Now if there is a body of mass M and whose volume is V, such that:

$\frac{M}{\sqrt[3]{\frac{3V}{4\pi}}} \geq 6.75 \times 10^{26}$--(5)

Equation (4) or equation (5) represents the case of either tremendous mass of the body or extremely small size of the body such that even light particles cannot escape from the surface of the body, then it is called the black body. This name is coined from the thermodynamics wherein the black body is that which absorbs the radiation of all visible ranges of spectrum. The black hole is the heaviest part of the Milky-Way. It is believed that when all the sources of nuclear fusion in a star are exhausted, the star dies and its dead body is the black hole. These black holes can swallow gas. They can modify the trajectories of nearby stars. They can also have a stable life close to a companion star. We can get the information about the black holes from their effects they render on the surrounding gas and stars. In the past few centuries the black hole physics has made many major breakthroughs in the field of astronomical sciences. Starting from Newton and Einstein, a large number of scientists working in this field have proposed their own theory to explain the observed phenomena in the galaxy, but still the black holes are a puzzle among the scientists. The Indian–American astrophysicist Subrahmanyan Chandrasekhar first speculated in 1930 that the massive stars just cool down and collapse into something denser such as white dwarfs and neutron stars. In 1974, the British theoretical physicist Stephen Hawking discovered that quantum black holes are not as black as classical black holes. He applied quantum theory and found that the black holes can emit thermal radiations also.

The black holes are based on very simple theory to describe. Three parameters are sufficient to characterize them fully. These are mass, angular momentum and electric charge. Astrophysical black holes are even simpler. They have no charge. If they are formed from a rotating collapsing star or from the merger of two neutron stars they must have conserved their rotational energy. So, the black holes may be found in rotating state also. Moreover, they can spin up due to their successive interactions. It was once thought that black holes do not emit any radiations, Stephen Hawking discovered that if quantum effects are taken into account, then it can be shown that the black holes can radiate thermal energy and particles. In this way the black hole may shrink until it evaporates. This effect is important for very tiny black holes, but astrophysical black holes are very massive and they would need a time much longer than the age of the Universe to evaporate.

2. Aspects of black holes: The Black hole has several dimensions for their study. It makes the center of rotations for all the astronomical bodies present in the solar systems.

2.1. Building block of a solar system: First of all, it may be taken as the basic building block of the evolution of the universe. Primordial black holes may have existed during the early stages of the Universe, a few moments after the Big Bang. At that time the temperature and pressure were of such a magnitude that they could have squeezed down to a singularity: however very few primordial black holes of about a billion tons of mass could have survived till date. The lighter black holes must have been evaporated. After the big bang the black hole came in existence first and then the other family members of the universe were formed which were revolving in various orbits about the black hole. So, every galaxy has a black hole around which every element of the Milky Way exists in their stationary orbits. If any how any one of the astronomical bodies breaks away its stationary orbit, it is engulfed by the black hole present at the center of the galaxy. In this way we can say that the whole galaxy has its existence due to the presence of black hole.

2.2. Death state of a living star: The death of a living star is most commonly known as the stellar collapse. There is struggling between the gravity and pressure forces within the star throughout its whole life. Gravity wants to compress the star, while the radiation due to nuclear fusion wants to pushes the matter outwards. When anyhow the fuel of the star is exhausted, the star stops burning. If the mass of the star is not comparable to that of our own sun, it will become fainter. It will cool down as a white dwarf. However, if its initial mass is about eight times greater than the Sun, it will collapse and it will bounce back and detonate in a supernova. in this way. Neutron stars are formed. And if the original mass of the star exceeds about 25 times the mass of the sun masses, then at the end nothing can counteract gravity and the whole mass is squeezed into a point of zero volume and infinite density. This point is called a singularity. The singularity is not naked rather it is surrounded by a boundary, called an event horizon. Hence what is left at the explosive end of the life of a very massive star is a singularity surrounded by an event horizon, which is called the black hole.

2.3. Rotational conjugate with other stars: This is also the basis of Stellar-mass black holes. According to galactic evolutionary models a billion stellar-mass black holes exist in our galaxy. Their masses are estimated to be between three and twenty times that of solar masses. There are some black holes which are in isolated form while there are some black which are lying close to normal stars. Such coupling of black holes and star is called black hole X-ray binaries or they are called the micro quasars [1-3]. They emit a relativistic pair of jets. The bright X-ray emission takes place due to the gas which is sucked from the normal star and spirals towards the black hole's event horizon.

Super massive black holes: The core of many galaxies is exceedingly luminous of approximately all wavelengths. This most bright area around the black hole is called Active Galactic Nuclei (AGN). The AGN extend only over a few light minutes or light-days. They are less than one ten-millionth the size of their host galaxy. They are very much brighter than the whole galaxy. They are super massive black holes. They are not isolated rather they are located at the center of galaxies. They attract the matter in their vicinity with their strong gravitational field. This fast attraction is termed as Accretion. The gas rotates towards the event horizon forming an accretion disc. While the gas spirals in, its energy is converted into heat and it starts to shine brightly. If a Part of the matter escapes off of being gulped down into the black hole are being carried away in two highly collimated radio jets that emerge close to the inner edge of the disc.

In the Milky way of our solar system there is not so much matter around the AGN, so the black hole is starving, so it is much less bright. This galaxy is less active [4]. The motion of the stars in its vicinity provides the best empirical evidence for the existence of a super massive black hole

2.4. Microscopic black holes: The lightest black holes are called elementary black holes, because they are similar to the elementary particles. These microscopic black holes may be produced due to collisions of hydrogen or helium particles. With the startup of the Large Hadron Collider (LHC) these microscopic black holes received a lot of attention as they might be produced during particle collisions at the LHC. But they must have immediately disintegrated again, so they have no enough time to accrete matter and cause any macroscopic effects. Till date not a single theory has become capable enough to explain all the observed properties of the black holes. However, the thermal properties of the black holes most probably would be explained by applying the black hole thermodynamics [5,6]. The black hole mechanics is given in four parts known as four laws of black hole thermodynamics.

The zeroth law of black hole mechanics states that the surface gravity K of a stationary black hole is constant over its event horizon This is analogous to the zeroth law of thermodynamics according to which the temperature T of a system is constant when it is in thermal equilibrium with another system. The first law of black hole thermodynamics also obeys the conservation of energy of the black hole by relating the change in the black hole mass M to the changes in its area A, angular momentum J and electric charge Q. It can be expressed as:

$$\delta M = \frac{1}{8\pi} K \, \delta A + \Omega \delta J + \Phi \, \delta Q \text{--} (6)$$

The first law of black hole thermodynamics implies that the surface gravity K, the angular velocity Ω and the electrostatic potential Φ all are constant over the event horizon of any stationary black hole. In this way we see that the first law of thermodynamics and the first law of black hole thermodynamics both are identical. The second law of black hole mechanics is also known as Hawking's area theorem [7]. According to it the area A of a black hole horizon cannot decrease. This is also similar to the second law of thermodynamics, which says the entropy S of a closed system cannot decrease. The third law of black hole mechanics states that the surface gravity K cannot be reduced to zero by any finite sequence of operations [8]. This is analogous to the Nernst theorem of law of thermodynamics. This is also called the third law of thermodynamics. It states that the absolute temperature T of a system cannot be reduced to absolute zero in a finite number of operations. Thus the four laws of black hole mechanics are similar to the four laws of thermodynamics. According to Bekenstein [9], the entropy of the black hole S is proportional to the event area A of the black hole. The temperature T is analogous to the black hole surface gravity K. Hawking made the remarkable discovery that black holes are not completely black but they also emit radiation [10, 11]. It was also found that the black hole radiation had a thermal spectrum. There upon Hawking realized that Bekenstein's idea is consistent and the black hole entropy is proportional to area of the event horizon. According to Hawking the black hole temperature is $T = K. /(2\pi)$, and $\eta = 1/4$. In his way we get the famous Bekenstein–Hawking formula for the entropy of a black hole:

$$S_{bh} = S_{BH} = \frac{A}{4} \text{--} (7)$$

Here the subscript bh stands for ‘black hole,’ and the subscript BH stands for ‘Bekenstein–Hawking. It was observed that the emission from a black hole persisted even when the black hole became effectively static. He pointed out on heuristic grounds that a rotating black hole should amplify certain waves and there should be an analogous quantum effect of spontaneous radiation of energy and angular momentum. Later Misner [12] and Starobin sky [13] confirmed the implication by a Kerr hole of scalar waves in the ‘super radiant regime. It is the region where the angular velocity of the wave fronts is lower than that of the waves. Bekenstein showed that implication should occur for all kinds of waves with positive energy density [14]. The argument for this spontaneous radiation was that in a quantum analysis the amplification of wave is stimulated emission of quanta, so that even in the absence of incoming quanta one should get spontaneous emission by using the relation between the Einstein coefficients for spontaneous and stimulated emission. Hawking calculated that there is emission of thermal radiation from the black holes and this was verified by several scientists working in this field [15].Kerr metric l gives not only the spontaneous but also the thermal emission also. Hawking [16] have calculated the density matrix of the emitted particles and found that it, as well as the expected number in each mode, is precisely thermal. The thermal emission from a black hole has been derived in a variety of ways by several people, so its prediction seems to be a clear on sequence of our present theories of quantum mechanics and general relativity.

3. Conclusion: Black holes are perhaps the most highly thermal objects in the universe, though they are very cold for stellar mass black holes. Their phenomenological thermodynamic properties are very well understood, when they are stationary. However, most of the characteristics of their microscopic degrees of freedom are not well known. It may be assumed that black holes are like other information storage devices which keeps all the information about the universe extended around them. So, there are a lot to know about the black holes. There is need to study about where and how the microscopic degrees of freedom store the information. Therefore, although we have gained an enormous amount of information about black holes and their thermal properties in the past 30 years, it seems that there is even much more that we have yet to learn about the black holes.

REFERENCES:

[1] Gillessen, Stefan, et al. 2009, “Monitoring Stellar Orbits Around the Black Hole in the Galactic Center”, The Astrophysical Journal, 692, 1075-1109

[2] Ghez, Andrea M. et al. 2008, “Measuring Distance and Properties of the Milky Way’s Central Supermassive Black Hole with Stellar Orbits”, The Astrophysical Journal, 689, 1044-1062

[3] Remillard, Ronald A. & McClintock, Jeffrey E. 2006, “X-Ray Properties of Black-Hole Binaries”, Annual Reviews of Astronomy and Astrophysics, 44, 49-92.

[4]Springel, Volker, Frenk, Carlos S. & White, Simon D. M. 2006, “The Large- Scale Structure of the Universe”, Nature, 440, 1137-1144.

[5] Bardeen J M, Carter B and Hawking SW 1973 Commun. Math. Phys. 31 161.

[6] Carter B 1973 Black Holes ed C DeWitt and B S DeWitt (NewYork: Gordon and Breach) p 57.

[7] Hawking SW 1971 Phys. Rev. Lett. 26 1344.

[8] IsraelW 1971 Phys. Rev. Lett. 57 397.
[9] Bekenstein J D 1972 PhD Thesis Princeton University, Princeton, NJ.
[10] Hawking SW 1974 Nature 248 30.
[11] Hawking SW 1975 Commun. Math. Phys. 43 199.
[12] Misner CW 1972 Bull. Am. Phys. Soc. 17 472.
[13] StarobinskyAA 1973 Zh. Eksp. Teor. Fiz. 64 48 (Engl. transl. Sov. Phys.—JETP 37 28).
[14] Bekenstein J D 1973 Phys. Rev. D 7 949.
[15] Boulware D G 1976 Phys. Rev. D 13 2169.
[16] Hawking SW 1976 Phys. Rev. D 13 191.

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APPENDIX 2

SOME PHYSICAL ASPECTS OF BLACK HOLES AND THEIR THERMAL-STATES

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Abstract: Presently, intensive study on the shape and size of black holes is going on among the scientists working in the field of astrophysics. Every week some new characteristics regarding the black holes are found published in different esteemed journals. In this paper some simple laws of physics have been proposed, which supports the formation of black holes. Some characteristics of the black holes reported in literatures have also been explained easily on this proposition.

Key words: Gravitational pull, White dwarf, black holes, Milky Way.

1. INTRODUCTION:

Black hole is an area in space from which nothing can escape from it, not even a light particle, (photons). This is due to the fact that the gravitational pull possessed by it is so strong that it does not allow anything to get escaped from it. The general theory of relativity predicts that a sufficiently compact mass can deform space time to form black hole. In many ways a black hole acts as an ideal black body as it reflects no light. When living star is exhausted of its fuel, the nuclear reaction responsible for its energy generation ceases. So, the continuous nuclear blast taking place within the star stops. Now, there is no internal force to encounter the gravitational pull in the star. Due to gravitational pull the size of the dying star goes on decreasing which in turn goes on increasing the gravitational pull. This process goes on multiplying in to the dying star, till the size becomes point like structure having no volume of its own. This state of the matter in space is being called as the black hole.

2. GRAVITATIONAL PULL OF CELESTIAL BODIES:

The gravity force on the surface of the earth is described by Newton's law of gravitation. On a mass of one Kg object the gravity force applied by the earth is given as: GM/ R^2. Where G is the gravitational constant. Its numerical value is given as: 6.63×10^{-11}Newton –meter^2 – Kg $^{-2}$. Here R is the radius of the Earth. Its numerical value is: 6400000 meters. M is the mass of the Earth, whose numerical value is: 5.972×10^{24}Kg. Substituting these values, it is found that the average value of gravity force acting on a body of one Kg mass is approximately 10 Newton. This type of force of attraction is exerted by each and every celestial body. This is also a fact that all the celestial bodies are of spherical shape. So, the gravitational force exerted by a celestial body not only depends upon its mass but also it depends upon the radius of the celestial body. The first factor is the mass of the body and the second factor is the radius of the spherical celestial body. The gravitational pull is directly proportional to the mass of the body and inversely proportional to the square of the radius of the sphere. Thus, the magnitude of this pulling force increases as the mass of the body increases. But this growth of pulling force does not increase linearly. The main cause of this decrease of this force is the increase of the radius of the celestial body with the increase of the mass of the body. If we consider the increase of mass only, then by increasing the mass 8 times, we find that the gravitational force increases by two times. But in the case of black hole, with increase of mass of the black hole its gravitational attraction increases very fastly. The root cause of this tremendous increase in the pulling force is that with the increase of mass of the black hole its size also decreases due to its own gravitational pull. With the increase of mass of the body the size of the body begins to decrease due to internal pulling forces. This becomes the root cause of the formation of the black hole. The cohesive force or the adhesive force acting between two molecules or atoms also goes on increasing; this also helps the formation of the black holes under some suitable circumstances.

3. BLACK HOLE CHARACTERISTICS:

When we imagine a tremendous amount of matter packed so densely that its gravitational pull on any particle is so large that nothing can escape from its gravitational pull, neither moon nora planet and nor even a light particle (photon) [1,2]. That is why this state of body is called a black hole. From such body even there will be no reflection or refraction or scattering of light particles. Since from a black hole light cannot escape, so it is also impossible for us to sense the other bodies in the neighbourhood of the black hole directly through our instruments, no matter what kind of electromagnetic radiation we use. The key is to look at the whole's effects on the nearby environment. In this context we can say that when a star happens to get too close to the black hole. It pulls on the star and rips it to shreds. When the matter from the star begins to bleed toward the black hole, it moves faster, so it becomes hotter and it begins to glow brightly. In this way the ambient of the black hole can be studied and the photographs may be possible.

4. BLACK HOLE INITIATIONS:

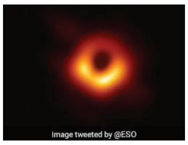
As it is reported that when a star several times greater than that of our sun goes on exhausted of its fuel such as hydrogen and other light gasses, the nuclear reaction of fusion which is the source of production of energy within the star diminishes. The internal pressure within the live star produced due the nuclear blast begins to diminish. It is called the dying star or white dwarf [3-5]. But it is still in unstable condition. The pressure inside the white dwarf goes on decreasing consequently; the gravity force goes on increasing. When the pressure inside it is overwhelmed by its gravity force and there is none to obstruct the gravity force within the star, then the equilibrium between gravity force and the pressure force is disturbed. Now the gravitational pull within the star goes on increasing and the size of the star also goes on decreasing. The remaining core collapses into a spot of infinite density and almost no volume. Now this state of the star is called the black hole [6-8]. It is reported that the smallest size of black holes ranges from the size of one atom having mass equivalent to masses of several mountains. The largest black hole is called super massive black hole. It is situated at the center of galaxies. They're each more than one million times more massive than that of the Sun. But how these beasts are formed is still being examined [9].

5.COALESCE PRINCIPLEOFCELESTIAL BODIES:

Generally, all the bodies in the nature are spherical in shape. If there are two celestial bodies being attracted towards each other due to gravitational pull, they will tend to form a single body. There may be several causes one of them may be their surface energy minimization. As for example, let us suppose that Mass of each body is M and radius is R. The surface energy of both the bodies in separate form will be = Twice of (4π R^2 S). Where S is the surface tension. When the two bodies will coalesce, the resultant surface area will become $=\sqrt[3]{4}$Times of (4π R^2S). It is apparent that this surface area is smaller than the surface area of the bodies when they are in separate status. Thus, it is natural that two or more white dwarfs will have tendency to be united into one form for minimizing their surface area, so that the surface energy will become minimum.

If we consider that the celestial bodies are moving around each other as in the case of other celestial bodies then their total energy will be sum of potential energy and kinetic energy. E= $\frac{-Gm1m2}{r}+\frac{mr^2}{2}\omega^2$. *Here* $m1$ *and*$m2$are the masses of two bodies , m is the reduced mass of the system, r is the separation between them.Since the potential energy is negative and the kinetic energy is positive .Hence there will must be a condition when the total energy will be optimum. To achieves this condition both the bodies will coalesce in the form of one body. In this way the probability of formation of black body takes place.

If we consider that the bodies such as white dwarf are at high temperature and they are in charged state. The particles in the white dwarf must be in concentric circular motions. Hence magnetic force of attraction will take place between the whirling particles of the white dwarfs. This force is responsible for minimizing the size of the composite bodies. This is the reason that the whirling motion of charged particles appear in the neighbourhood of the black holes.



Black hole photograph from literature.Science | Edited by Niharika Banerjee | Saturday April 13, 2019, the first-ever black hole to be photographed has been named "Powehi".

The name Powehi, meaning embellished dark source of unending creation, was deliberated upon by astronomers and renowned Hawaiian language professor, Larry Kimura.

The whirling motion of charged particles in black holes, copied from literature.

Near black hole the bodies are attracted with tremendous force of attraction. Due to perturbation by other bodies the motion becomes centripetal in nature. Some bodies make direct collision with heavier body. Under certain circumstances some mass of the body is converted into energy of high frequencies. Some of which are in visual range. So, in the neighbourhood of black hole some light appears.

6. ISOLATION OF BLACK HOLES:

It is estimated that once a black hole is formed, in the range of 20000 light years there will be no other bodies found. These bodies will be swallowed by the black hole. Even light particles cannot pass through the neighbourhood of the black hole.

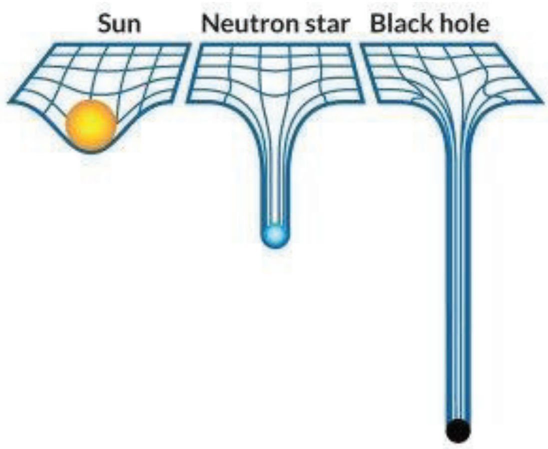
It is safe to observe a black hole if we stay away from its event horizon. It is similar to the gravitational field of a planet. This zone is the point of no return, when we are too close to the back hole there will be no any hope of rescue. But we can safely observe the black hole from outside of this arena.

7. DIFFERENT STAGES OF STARS:

When a white dwarf is formed, initially its temperature is very high. But because it has no source of energy, it would gradually cool as it radiates its energy. Therefore, that its radiation, which initially has a high colour temperature, will lessen and redden with time. A white dwarf takes very long time to cool. Then it begins to crystallize starting with core. Therefore, a star at low temperature will no longer emit significant heat or light, and itis called a cold black dwarf. Because the length of time it takes for a white dwarf to reach this state is calculated to be longer than approximately 13.8 billion years. So, it is thought that no black dwarfs yet exist. The oldest white dwarfs still radiate at temperatures of a few thousand Kelvin's.

It is estimated that a supernova may occur about once in every 50 years in a galaxy of the Milky Way [10,11].An star can always be exploding somewhere in the universe. Some of those aren't too far from Earth. About 10 million years ago, a cluster of supernovae created the "Local Bubble," a 300-light-year long, peanut-shaped bubble of gas in the interstellar medium is present that surrounds the solar system. However, it depends in part on its mass. Presently our sun doesn't have enough mass to explode as a supernova. However, if once the sun will become out of its nuclear fuel, perhaps in a couple billion years, it will swell into a red giant that will likely vaporize our world, before gradually cooling into a white dwarf [12]. But with the right amount of mass, a star can burn out in a fiery explosion [13,14].When an old star is about to die it's volume goes on shrinking keeping the mass the same which increases its density. Now this creates a very large depression in the body creating a very strong gravitational pull. Finally, it may form a black hole. In black hole millions and billions of tons of mass is packed in a relatively small volume which creates a point of almost infinite density leaving a "hole" in the structure.

White



Just like this but in 3-D. This depression is so deep that its gravitational pull is also almost infinite. This force of gravitation is so strong that the escape velocity even exceeds the velocity of light. In this figure the relative strength of gravitational pull has been shown in the living star, in the white dwarf and in the black hole.

8. CONCLUSION:

Thus, a black hole is a point of infinite density on the space time curvature. The formation of black hole starts from a star which is a matter in space and it ends in nothingness in space. Black holes are also said to be get way between different dimensions under some defined circumstances of a minimum size of stellar mass. The chain of fusion reactions taking place into the star comes to an end. Therefore, the remaining stellar material collapses under its own gravity to a very dense remnant. This has been very well validated in the photographs of the event horizon of a black hole recently published as the first direct observational evidence of a black hole.

References:

[1] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov and S. Zerbini, JCAP 0502, 25 (2005).

[2] A. Cooney, S. DeDeo and D. Psaltis, Phys. Rev. D 82, 064033 (2010).

[3] J. Q. Guo and A. V. Frolov, Phys. Rev. D 88, 124036 (2013).

[4] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. 82, 451 (2010).

[5] M. Calza, M. Rinaldi and L. Sebastiani, Eur. Phys. J. C 78, 178 (2018).

[6] S. Capozziello, S. J. G. Gionti and D. Vernieri, JCAP 1601, 015 (2016).

[7] S. S. H. Hendi, R. B. Mann, N. Riazi and B. Eslam Panah, Phys. Rev. D 86, 104034 (2012).

[8] S. Capozziello and M. D. Laurentis, Phys. Rep. 509, 31422 (2015).

[9] D. Kubiznak and R. B. Mann, J. High Energy Phys. 1207, 033 (2012).

[10] N. Altamirano, D. Kubiznak, R. B. Mann, and Z. Sherkatghanad, Class. Quantum Gravit. 31, 042001 (2014);

[11] A. M. Frassino, D. Kubiznak, R. B. Mann and F. Simovic, JHEP 1409, 080 (2014);

[12]A. Belhaj, M. Chabab, H. El Moumni, K. Masmar, M. B. Sedra and A.

[13] S. H. Hendi, Z. S. Taghadomi and C. Corda, Phys. Rev. D 97, 084039 (2018).

[14] S. H. Hendi, R. B. Mann, S. Panahiyan and B. Eslam Panah, 95, 021501(R) (2017)

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APPENDIX 3

ENIGMA OF SUPER MASSIVE BLACK HOLES

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Abstract:

The entire galaxies and solar systems are full of enigmatic elements with their complex behaviours. Supper massive black holes are one of the central bodies around which the galaxies exist. The tremendous gravitational attraction of the black hole does not allow any particle to pass through the black holes. It is engulfed by the black hole. Even light particles are not allowed to pass through the black holes. It is also documented that in each galaxy there are so many black holes, which seems to be contradictory statements to each other. It has become a matter of interest how it is possible. This puzzle may be allowed to hold in the universe if explained on the basis of Carnot's ideal heat engine.

Key words: Supper massive black holes, galaxies, Carnot's ideal heat engine.

1. INTRODUCTION:

The formation of stellar-mass black holes is well documented. But it is not clear how black holes having masses of a million to a billion times larger the mass of the Sun could have been formed. It is also investigated that the Super massive black hole cannot be formed through only the collapse of a single star. There must be some other processes for the formation and growth of super massive black holes. The investigation of such processes has become the interest of intense study in the present era. One possible scenario for the formation of super massive black holes is that the black holes of intermediate masses merged to form a more massive black hole. But this type of interaction must be allowed under certain constraints, otherwise due to persistence of such attractive interaction the whole universe will converge into a single entity of black hole. The mass of a black hole can be determined by studying the material that orbits around it. This is one of the most common techniques for measuring the masses of black holes.

The best evidence for the existence of a super massive black hole is provided by the center of our own galaxy. The movements of the individual star at the Galactic Centre have been interests of study by a number of astronomer's foe more than two decades. To present such model of orbits of the stars there will be need of an unseen mass of four million times the mass of the Sun located near the center of the orbits. We have discussed here a method to produce work by running a heat engine between two black holes which serve as thermal reservoirs. In accordance with the Carnot's heat engine it can be concluded that the two black holes can ultimately merge into one bigger black hole keeping the total entropy unchanged. The resulting black hole thus must have a smaller mass than the total mass of the input black holes. The difference corresponds to the extracted work. It can be shown easily that by merging two black holes with masses M_1 and M_2we can extract up the useful work. Black hole thermodynamics has been discussed extensively since the seminal papers by Bekenstein [1] and Hawking [2,3]. However, not much attention has been paid to heat engine methods. A model suggested by Kaburaki and Okamoto [4] uses a Kerr black hole as the working medium in a Carnot-like engine running between two reservoirs formed by boxes with radiation. Recently Deng and Gao proposed a Carnot engine with radiation as the working fluid and a black hole as the cold reservoir [5]. In this paper, we have presented a simplified model of heat engine that allows the bigger black hole to absorb the light black hole which is responsible for the formation of super massive black holes by applying the principle of ideal Carnot's ideal heat engine.

2. BLACK HOLERADIATION IN A BLACK BOX:

A non-rotating and uncharged black hole packed into a box is considered which is filled with radiation [10]. The inner wall of the box is assumed to be perfectly reflecting. Thus, the box is in isolation from the outside world. Therefore, the total energy in the box is constant. In this way, the black hole is taken to being thermodynamic equilibrium with the radiation. The mechanism of black hole radiation has been described by Hawking [1-3]. In this model the radiation is assumed to be entirely due to photons. The entropy of the black hole may be given as [1].

$$S_{BH} = 4\pi K\left(\frac{M}{M_p}\right)^2 \text{---}(1)$$

Where k is the Boltzmann constant is the mass of the black hole and M_P is called the Planck's Mass: $M_P=\sqrt{\frac{\hbar C}{2\,\pi\, G}}.= 2.2 \times 10^{-8}$ Kg. G is the Gravitational constant, Cis the velocity of light, h is the Planck's constant.

The real composition of the radiation would influence the numerical values of the parameters. The total energy of the system is given as:

$$E_{total}= MC^2+aVT^4 \text{---}(2)$$

where the first term is the black hole energy and second term is the energy of the radiation:

$$a=\frac{\pi^2 k^4}{15\, c^3 h^3} \text{---}(3)$$

V is the volume of the box and T is the radiation temperature. K is Boltzmann constant, c is velocity of light, and h is the Planck's constant. The black hole temperature is given as [7]:

$$T_{BH} = \frac{\partial E_{BH}}{\partial S_{BH}} = \frac{\hbar c^3}{8\pi kGM} \text{---}(4)$$

The entropy of the combined system is given as:

$$S_{total} =S_{BH} + S_{rad} \text{---}(5)$$

The radiation entropy is given as [2]

$S_{rad} = \frac{4aVT^3}{3}$ ---(6)

By using Eq. (2), we can express the radiation temperature in terms of the total energy and the black hole mass such that the total entropy is a function of M. The entropy of the system will be composed of a black hole and radiation inside a box with insulating walls on the mass of the blackhole.

3. CARNOT PROCESS BLACK BOX:

The model is shown in Fig. (1) There are two boxes, each with radiation and a black hole in stable equilibrium. Each box serves as a heat reservoir, with its temperature determined by the mass of the black hole. Between the boxes

there is a connecting cylinder with a movable piston. The volume of the cylinder is much smaller than the volume of the reservoirs. The cylinder walls as well as the piston are assumed to be perfectly reflecting. The cylinder can either be completely isolated or it can be open to one of the reservoirs.

A is big black hole in the box A' in which there is cold radiation and B is small black hole in the box B' in which there is hot radiation. (a) Radiation from the hot reservoir enters the cylinder and pushes the piston, so that the radiation in the cylinder expands isothermally. (b) The cylinder is isolated and the radiation expands adiabatically by pushing the piston and cooling to the temperature of the cold reservoir. (c) The cold radiation is pushed out of the cylinder Isothermally into the cold reservoir.

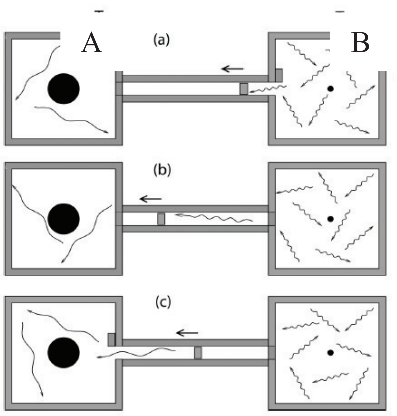

Fig. 1. Components of ideal Carnot's heat engine in terms of black hole box.

4. WORKING OF THE BLACKHOLE HEAT ENGINE:

The P -V diagram of the cycle is shown in Fig. 2. The cycle starts with the piston close to the hot reservoir, and the volume of the working medium is zero. (a) In the first step, the cylinder is open to the hot reservoir and the piston moves to increase the volume of the working medium to Va. The process is isothermal and isobaric. The radiation pressure is given by:

$P = \frac{aT^4}{3}$ ---(7)

Thus, the radiation pressure is uniquely determined by the temperature. The work done by the system during this step is given as:

$$W_a = P_1 V_a = \frac{aT_1^4 V_a}{3} \text{-----}(8)$$

where p_1 and T_1arethe pressure and temperature of the hot reservoir. The energy extracted from the reservoir is given as:

$$Q_a = T_1 \Delta S_a = \frac{4aV_a T_1^4}{3} \text{-----}(9)$$

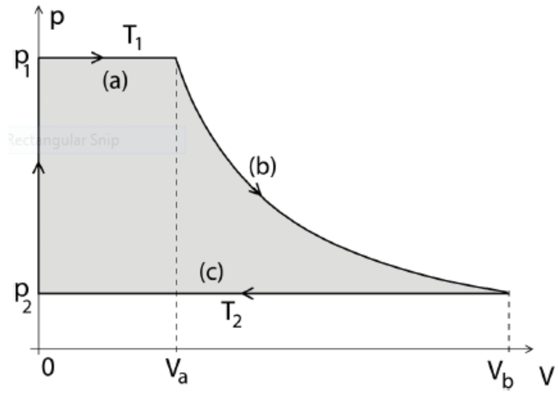

Fig. 2.A P-V diagram of the cycle.

The process consists of (a) isothermal and isobaric expansion, (b) adiabatic expansion, and (c) isothermal compression.

(b) In the second step, the cylinder is isolated and the radiation expands adiabatically by pushing the piston to volume V_b, cooling to the temperature T2. Because the radiation entropy given by Eq. (7) remains constant, the resulting volume is $V_b = [V_a \{\frac{T_1}{T_2}\}^3]$and the work done by the system in this step is given by:

$$W_b = a\, T_1^4 V_a\left(1 - \frac{T_2}{T_1}\right) \text{-----}(10)$$

(c) In the third step, the cylinder is opened to the cold reservoir and the radiation is isothermally pushed out. Because the radiation is pushed to the region with non-zero pressure, work must be performed on the system. The work is given by:

$$W_c = -P_2 V_b = -\frac{1}{3} a\, T_1^3 T_2 V_a \text{-----}(11)$$

And the energy transported to the cold reservoirs is given as:

$$Q_c = T_2 \Delta S_c = \frac{4}{3} a\, V_b T_2^4 = \frac{4}{3} a\, V_a\, T_1^3 T_2 \text{-----}(12)$$

(d) In the last step, the piston is carried through the empty cylinder to its initial position. No working medium is involved, so that no energy exchange or work occurs. The network in one cycle is therefore, W = Wa + Wb + Wc

W= $\frac{4}{3}$ a Va T_1^3(T1-T2) --(13)

W= Qa−Qc---(14)

In this way it is found that Carnot's Equation is satisfied.

Therefore, $\frac{Q_a}{Q_C} = \frac{T_1}{T_2}$---(15)

Therefore, the efficiency of the engine may be written as:

$\eta = \frac{W}{Q_a} = 1 - \frac{T_2}{T_1}$---(16)

5. ENERGY EXTRACTION:

In each Carnot's cycle energy is extracted from the hotter reservoir and deposited to the cooler reservoir. The properties of the reservoir will also change with time. This change can be expressed by Eq. (14) with:

Qa = $-dM_1C^2$ and Qc = dM_2C^2, which means that the energy transport is compensated by the decrease or increase of the black hole mass to find a new equilibrium with the radiation. Thus, we obtain

$\frac{dM_1}{dM_2} = -\frac{T_1}{T_2}$--(17)

For a black hole $T \propto \frac{1}{M}$ Therefore, we can write $M_1 dM_1 = M_2 dM_2$

$$M_1^2 + M_2^2 = M_{1,0}^2 + M_{2,0}^2, \qquad (18)$$

Where $M_{1,0}$ and $M_{2,0}$ are the initial masses of the black holes. We can find this result directly by observing that during the reversible processes the total entropy does not change. During this process the hotter reservoir loses mass and becomes hotter and the colder reservoir gains mass and becomes colder. This behaviour is counter intuitive. The temperatures of two bodies in thermal contact converge. Since a black hole has a negative heat capacity, therefore, its temperature increases when energy is taken from it. In theory the black hole of the hot reservoir will ultimately disappear. At the end point the cold black hole will survive with total mass of the system.

$$M_f = \sqrt{M_{1,0}^2 + M_{2,0}^2}. \qquad (19)$$

The total work extracted during the entire process is equivalent to the decrease of the mass,

We illustrate this situation on the energy-entropy diagram in Fig. 3. The shaded area represents the available states of the system consisting of black holes. The boundary S∝ E2 represents states with a single black hole. In the equilibrium states for the given energy the entropy is maximum. If we start the initial state in which the state is not in equilibrium state as Aof two or more than two black holesthen , equilibrium can be reached bydifferent paths. Path AB_1represents the merging of the two black holes when energy is kept fix. Therefore, through this path no work is being done. In this state entropy is maximized with the available mass. Path AB_2 represents a collision of two black holes: a single black hole is produced and part of the energy is carried away by the gravitational waves. This energy is in the range of 10^{-3}–10^{-2} of the rest energy of the input black holes [8]. Path AB4 corresponds to the reversible process described in this section for which the entropy is unchanged and the maximum possible work is extracted. Path AB3 represents a slightly more realistic version of this process. Because due to imperfections the entropy will increase and less work will be gained.

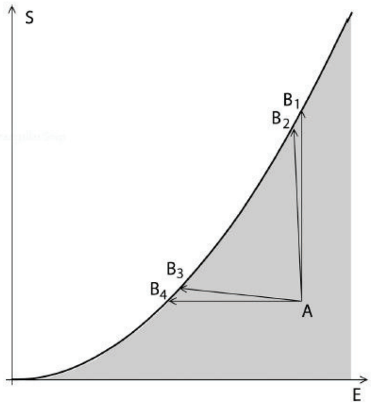

Fig. 3. Energy and entropy of a system of black holes.

The shaded area represents no equilibrium states with several black holes in the system, and the boundary $S \propto E^2$ represents equilibrium states with maximum entropy. When system A is in equilibrium state it can attain equilibrium by several paths. A difference in the E-coordinate represents the work gained from the system.

6. POWER:

The Carnot cycle is idealized because it assumes only the reversible processes, which are infinitely slow. In real processes a compromise is made between the requirements for the maximum efficiency and the maximum power. In the black hole process we have discussed, the power of the engine will be limited by the rate at which the source of heat can supply the heat energy and the sink can absorb energy, which is given by the radiation power of the black holes. The power of the unused amount of heat, the difference of the two heats is converted into work. The amount of work done per second is called the power. To maintain the thermal equilibrium energy would be released at slower rate from the hot reservoir than that of the absorbed by the black hole of larger mass at lower temperature.

7. BLACK HOLE ECONOMY:

If there are two black holes the first possibility is that they would combine together and there will be gain of useful work and finally a bigger black hole will be created. The second possibility is that the two black holes would be evaporated, some work will be accomplished and some radiation would be produced. The latter option would perform the work faster, but in the end, there would be no black hole left in this phenomenon. If we assume that thermal radiation is the ultimate waste, the former option is environmentally friendlier. In this sense, there will be gain of energy by cleaning the mess out of the universe. While growing the holes, we can also extract useful work, but it is possible at much slower pace than that of with evaporating the holes. But, provided that the expansion of the universe continues, and the background radiation cools down, all the black holes will ultimately be hotter than the background radiation and no more work can be obtained by feeding the background radiation to the holes. Therefore, by merging two black holes and getting energy out of the radiating the waste energy to empty space would become the only source of useful energy available.

8. DISCUSSION AND CONCLUSION:

In this proposition we have not considered the role of external radiation outside the boxes and the cylinder. It may be possible only when it is assumed that the universe outside the system is empty. Again, in this process we shall have to assume that this process will never come to a stop point. Because in the long run the bigger black hole will absorb the smaller one, then what will happen to the smaller black hole? Or what will happen to the black hole engines? But if we assume that the process is natural and sustainable then such question will not arise. If it can be found that the larger black hole is continuously increasing in mass the theory of black hole engine will get support. The universe will sustain till the super massive black hole is increasing its mass. For this it is also necessary that in the influence of supper massive black there must also be black holes of smaller masses. Thus, in each solar system there will be several black holes having their masses abnormally different. The heat exchange between the reservoir and the engines must be sufficiently slow so that the reservoir has time to reach thermal equilibrium via the interaction of the black hole with the field.

REFERENCES

[1] J. Q. Guo and A. V. Frolov, Phys. Rev. D 88, 124036 (2013).

[2] S. W. Hawking, “Black holes and thermodynamics,” Phys. Rev. D 13,191–197 (1976).

[3] A. Palatini, Rend. Circ. Mat. Palermo. 43, 203 (1919).

[4] O. Kaburaki and I. Okamoto, “Kerr black holes as a Carnot engine,” Phys.

Rev. D 43, 340–345 (1991).

[5] S. S. H. Hendi, R. B. Mann, N. Riazi and B. EslamPanah, Phys. Rev. D 86, 104034 (2012).

[6] W. Hu and I. Sawicki, Phys. Rev. D 76, 104043 (2007).

[7] S. W. Hawking, “The quantum mechanics of black holes,” Sci. Am. 236

(1), 34–40 (1977).

[8] Deng Xi-Hao and Gao Si-Jie, “Reversible Carnot cycle outside a black

hole,” Chin. Phys. B 18, 927–932 (2009).

[9] S. Capozziello, S. J. G. Gionti and D. Vernieri, JCAP 1601, 015 (2016).

[10] D. N. Page, “Particle emission rates from a black hole: Massless particlesfrom an uncharged, nonrotating hole,” Phys. Rev. D 13, 198–206 (1976).

[11] B. R. Parker and R. J. McLeod, “Black-hole thermodynamics in an undergraduatethermodynamics course,” Am. J. Phys. 48, 1066–1070 (1980).

[12] P. S. Custodio and J. E. Horvath, “Thermodynamics of black holes in a finitebox,” Am. J. Phys. 71, 1237–1241 (2003).

[13] P.Anninos, R. H. Price, J. Pullin, E. Seidel, and W.-M.Suen, “Head-on

collision of two black holes: Comparison of different approaches,” Phys.

Rev. D 52, 4462–4480 (1995).

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APPENDIX 4

Study of thermodynamic fluctuation theory with application to some critical phenomena in black hole

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Abstract:

There have been found some critical behaviors, like scaling power laws, divergence of fluctuations etc. in the black holes, which have certain intricacies in interpreting these notions. Using the thermodynamic fluctuation theory it is seen that divergences at a continuous turning point of the thermodynamic function indicates a change in stability. This gives some the relation between fluctuations and critical phenomena. It is found that the extremal phase of the black hole corresponds to a critical point of a continuous phase transition. The formalism of fluctuation theory can be extended to give a geometric interpretation of some of the usual critical behavior of black holes. Certain algorithms may exist between the critical behaviours and the thermodynamic fluctuation taking place in the black holes which needs to be explored in this article.

Key words: Thermodynamic- fluctuation, critical behaviors, black holes

1. Introduction: Those physical quantities which describe a macroscopic body in the state `of equilibrium are almost always close to their mean values. However, there are always certain deviations in the physical quantities from their mean values. This is the natural behavior of the macroscopic system. These deviations are known as thermodynamic fluctuations. The first statistical–mechanical theory of fluctuations was developed by Gibbs [1], But this work of Gibbs was not explored up to Einstein at that time. The Gibb's explanation of fluctuations was based on classical dynamics of particles in the micro-states defined in the phase space of position coordinates and momentum coordinates. Einstein's theory [2] is more general. The comparison of the Gibbs and Einstein approaches is found in Redound- Sukhanov article [3]. If we take the view that the thermodynamic fluctuation theory is to be relatively autonomous, then the simplicity of its internal logical structure is more important. Thus, we propose to build this theory on a fluctuation law which is postulated for an isolated system.

In the formation of matter a very important role is played by the scalar field, basically which has two components. The first component is the gravity field which is produced due to the mass of the matter and the second field is the pressure field. The effects of these two fields are opposite to each other. If initially the field is weak and the pressure is greater, then the pressure overcomes the field and the matter ball expands away leaving nothing behind it. If the gravity field is strong enough to overcome the pressure field, then the ball collapsed to form a black hole. But there may be a third case such that the field cannot decide whether to evaporate away or collapse to form a black hole on the other hand it begins to oscillates rapidly performing a large number of oscillations in a finite time. This frame shows the scalar field as a function of position. The gravitational field changes the frequency of the field. This produces characteristics colors to be found in the black hole "Black Holes of nature are the most perfect macroscopic objects that are in the universe. since the general theory of relativity gives only a single unique family of solutions for their descriptions, they are the simplest objects as well"-This is a very beautiful explanation n of black holes in the book, "The Mathematical Theory of Black Holes" given by S. Chandrashekar. But it was the revelation of Hawking, that black holes emit radiation, while they were considered only to absorb objects. These phenomena may be explained on the basis of the third case of both the fields forming the matter. It became very clear when Beckenstein showed that the temperature of the radiation can be attributed as the temperature of the black hole itself. While the fact that black holes are described by a set of very minimal number of fundamental quantities like mass, spin and charge, it is very puzzling that it be attributed with a thermodynamical concept of temperature. We know that the very reason we have thermodynamically description of matter, is due to the macroscopic constituency. This puzzling contrast in the behavior of black holes stays unexplained even today, except for the proposed solutions of String theory. If we attribute equilibrium of thermodynamical functions to black holes, then it can tempt many researchers to proceed further and study fluctuations and phase transitions related phenomena in them. There may be found critical behaviors, like scaling power laws, divergence of fluctuations etc. in which there are certain intricacies in interpreting these notions in the context of black holes. Also, we do not have a complete microscopic picture of black holes as of now. This is the case that there has been attempts to apply methods of renormalization to the entanglement entropy of the black holes, which have been found to contribute in part to the entropy of Hawking radiation.

2. Entropy and thermodynamics: Entropy is a measure of disorder of the system. It was first of all introduced by R. Claussius in 1850 to find the amount of heat reversibly exchanged at constant temperature T. It establishes a relation between the thermodynamics and the statistical mechanics. It is the most extensive parameter of thermodynamics. The entropy is a part of the first law of thermodynamics as well as the second law of thermodynamics.

According to the first law of thermodynamics:

$\delta Q = \delta U + P \delta V + \mu \delta N$ -- (1)

But according to second law of thermodynamics: $\delta S = \frac{\delta Q}{T}$ ---------------------- (2)

From equations (1) and (2), We can write:

$T \delta S = \delta U + P \delta V + \mu \delta N$

$\delta S = \frac{1}{T}[\delta U + P \delta V + \mu \delta N]$ ---(3)

Equation (3) represents the differential form of the entropy. Here we can say that entropy is a function of (U, V, N). So, knowing this quantity the entropy of the system can be determined and hence all the thermodynamic parameters of the ideal gas can be determined. For an isolated system, where there is no exchange of heat, i.e., $\delta Q = 0$. Therefore, $\delta S = 0$. Hence entropy is maximum. But for irreversible process it can be shown that$\delta S>0$.Thus in thermodynamics the state of equilibrium may be defined as the state of the maximum entropy.

The entropy of an ideal gas may be written as:

$S\ (N, T, P) = N\ K_B\ [S_O\ (T_O, P_O) + \mathrm{Log}_e\ \{(\frac{T}{T0})^{5/2}\ (\frac{Po}{P})\}]$ ---------------------------- (4)

Where $S_O\ (T_O, P_O)$ is an arbitrary dimension less function of the state (T_O, P_O). It comes due to integration. Since $PV = NK_BT$ and $U = \frac{3}{2}\ NK_BT$. Thus equation (4) gives the complete information about the ideal gas. Further, the entropy of the ideal gas can also be written in the form of f(N,V,U) as given below:

$S\ (N, V, U) = N\ K_B\ [S_O\ (N0, VO, U_O) + \mathrm{Log}_e\ \{(\frac{N0}{N})^{5/2}\ (\frac{U}{U0})^{3/2}(\frac{V}{V0})\}]$ ------------- (5)

From this equation all the equations of the ideal gas can be obtained.

As for example:$(\frac{\partial S}{\partial U})_{NV} = \frac{3}{2}NK_BT$. -- (6)

And$(\frac{\partial S}{\partial V})_{NV} = N\ K_B\ T$.-- (7)

From statistical mechanics the entropy of the system is given by the natural logarithm of the number of microscopic states Ω which is read as:

$S = \mathrm{Log}_e \Omega$ --- (8)

Here the Boltzmann constant is taken as unity. The microstate Ω will be functionof the macro state i.e. $\Omega = \Omega\ (U, V, N)$. When entropy is a function of these variables' equation (8) is very important because it provides the basic relation between the macroscopic thermodynamic quantity entropy and the statistical microscopic physics i.e. number of microscopic states. From this equation it is apparent that when $\Omega = 1$ then $S = 0$. Thus, when there is only one microscopic state, there is no probability of disorder so there will be no entropy.

3. ENTROPY AND Fluctuation: The main problem is to deduce the probability distribution of these deviations. The equation No. (8) may be written as: $\Omega = e^S$. This is the starting point of the thermodynamic fluctuation. It was first of all done by Einstein. However, we preferably denote P as the probability distribution, i.e. $P \propto e^S$. The entropy about the fluctuation quantity x up to the second order may be written as $S(x) = S\ (0) - \frac{1}{2}\beta\ X^2$, where $\beta = \frac{\partial 2s}{\partial X2}$, and $S(o) = 0$ because S has maximum value, therefore,

$\partial S / \partial X$ is also zero. Therefore, $P(X) = Ae\ {-\frac{1}{2}}\beta\ X^2$

$P(X)\ dx = (Ae\ {-\frac{1}{2}}\beta\ X^2)\ dX$-- (9)

The constant A is given by the normalization condition,

$\int_{-\infty}^{+\infty}$ P(X) dx=1--- (10)

In this way the constant is found to be $\sqrt{\frac{\beta}{2\pi}}$By Gaussiun Integrationformula.There the probability distribution of the variousvaluesof Gaussian function may be written as:

P(X)= $\sqrt{\frac{\beta}{2\pi} e - \frac{1}{2}\beta X^2}$--- (11)

This probability distribution is recognized as a Gaussians distribution function.

It reaches a maximum value when X=0 and decreases slowly as X increases. The mean squared fluctuation is defined as:

$\langle X^2 \rangle$=$\int$ P(X)X^2dx=$\frac{1}{\beta}$-- (12)

So, the Gaussian distribution may be written as:

P(X)dx= $\frac{1}{\sqrt{2\pi(X^2}} e(\frac{-x}{2\langle X\rangle^2}^2)$dx-- (13)

It can be seen that the smaller the value of $\langle X^2 \rangle$, there will be sharper themaximum value of P(X), which is the characreristics of the Gaussiandistribution. Now, let us consider the Gaussian distribution of more than one variable. In this case we will determine simulteneouse deviation for several thermodynamic quantities from their mean values. We define the entropy S as a function of quantities of simultaneous deviations, S (x_1,x_2,x_3,……..x_n). With Taylor expansion method, it can be shown that

S-S_0= - $\frac{1}{2}\sum_{ij=1}^{p} \beta ij\ xixj$-- (14)

Here it is noted that βij = βji, for simplicity the summation sign is omitted, then the equation may be written as:

S-S_0= - $\frac{1}{2}$ $\beta ij\ xixj$--- (15)

The probability takes the form: P = A exp. (-$\frac{1}{2}\beta ij\ xixj$)----------------------- (16)

A is a normalization constant, whose value is determined

$\int P(x1, x2 \ldots\ldots xn)dx_1$ dx_2dx_3 ---- dxn = 1--------------------------------------- (17)

After some manipulations we find, A = $\frac{\sqrt{\beta}}{(2\pi)^{n/2}}$, Therefore the desired form of Gaussian distribution for more than one variable may become:

P =$\frac{\sqrt{\beta}}{(2\pi)^{n/2}}$exp. (-$\frac{1}{2}\beta ij\ xixj$)--- (18)

Whereβij- $\frac{\partial 2s}{\partial Xi2\ \partial Xj}$, β= $|\beta ij|$--- (19)

4. Thermodynamic Fluctuation: Fluctuations of thermodynamic properties play a crucial role in many physical phenomena and diverse applications. Fluctuations are especially important in nanometer-

scale systems where they can lead to large variability of mechanical and functional properties. Fluctuations are unavoidable in molecular dynamics and Monte-Carlo simulations of materials, where the system dimensions rarely exceed a few nanometers. In fact, in many atomistic calculations, equilibrium properties of interest are extracted by analyzing statistical fluctuations of other properties [4-6]. Presently, fluctuations are primarily discussed in the statistical–mechanical literature and typically in the context of specific models. At the same time, the thermodynamics community traditionally relies on classical thermodynamics [7-9] which, by the macroscopic and equilibrium nature of this discipline, disregards fluctuations and operates solely in terms of static properties.

5. THERMODYNAMIC GEOMETRY

It has been found that the thermodynamic fluctuation theory gave us insights into the stability and critical behavior of the system. Now the whole formalism can be put into the language of geometry as was done by Ruppeiner [10]. Here, we Consider a black hole and its environment or the 'universe'. The total entropy is given as :

$S_{tot} = S_{bh} + S_e$. Where S_{bh} = Entropy of the black hole, Se = Entropy of the environment or the universe.

The fluctuation in the parameter α may be defined as: $F_\alpha = \frac{\partial Sbh}{\partial X\alpha}$ Where X_α=is a function of (M, J, Q). Now expanding the total entropy to the second order in fluctuations gives:

The linear terms vanish due to conservation laws and for a very large environment the second quadratic term is negligible compared to the first. Therefore, the above equation can now be written as:

$$\frac{\Delta\, Stotal}{KB} = -\frac{1}{2} g_{\mu\upsilon}(\Delta X^{\mu})\,(\Delta X^{\upsilon}) \text{ -- } (20)$$

Where the symmetric matrix g is given by

$$g^{\alpha\beta} \propto \frac{\partial^2 S}{\partial X^{\alpha} \partial X^{\beta}} \text{ -- } (21)$$

From this we can develop a formalism of Thermodynamic Riemannian geometry by defining the line element as:

$$\Delta l^2 = -2\Delta Stot / Kb = g^{\mu\nu}\Delta X^{\mu}\Delta X^{\nu} \text{ -- } (22)$$

This unitless positive definite line element can be given a physical interpretation: Farther apart the stateless probable are the fluctuations between the states [11].

Once a matrix is defined, we can go ahead and calculate the curvature of the given geometry. The thermodynamic curvature obtained can be given various interpretations as the range of interaction, the correlation volume etc. in various contexts. Refer [10] and references therein for detailed expositions. For most of the ordinary thermodynamical systems the curvature is negative. But it is very interesting that it is positive for the Kerr-Newman black hole. Even more curiously it is also positive for a Fermi gas. Therefore, attempts may be made to establish a correspondence between a two-dimensional Fermi gas and a Kerr Newman black hole.

6. Discussion:

When black Holes are characterized by just 3 parameters of mass, charge and angular momentum, they are comparable to elementary particles having just mass, charge, spin. But when low energy systems are being complexly constituted by such elementary particles, they have shown amazing richness and contrast in their equilibrium thermodynamical and critical behaviours. Generally, black holes being ’simple’ objects, generally show such behavior similar to these complex systems. In studying the phase transitions of black holes, we encountered some intricacies in the interpretation of critical behaviours and fluctuations. The use of equilibrium fluctuation theory provided some insight information’s into these matters that reveal the critical behaviours of the black holes. It might be possible that further investigations into the field can lead to some information aboutthe hawking radiation related problems. There might be remote or close links between the critical behaviours of the black holes and the microscopic theory of gravity.

References:

[1] J. W. Gibbs, The Collected Works of J. W. Gibbs, vol.2, Yale University Press, New Haven, 1948, pp.1–207.

[2] A. Einstein, Ann. Phys., Lpz.33(1910)1275–1298.

[3] Y. D. Rudoi and A. D. Sukhanov, Phys.Usp.43(2000)1169–1199.

[4] D. Frenkel, B. Smit, Understanding Molecular Simulation: from Algorithms to Applications, second ed., Academic, San Diego,2002.

[5] D.P. Landau, K. Binder, A Guide to Monte Carlo Simulations in Statistical Physics, third ed., Cambridge University Press, Cambridge,2009.

[6] J.J. Hoyt, Z.T. Trautt, M. Upmanyu, Math. Comput.Simul.80(2010)1382–1392.

[7] J. W. Gibbs, The Collected Works of J. W. Gibbs, vol.1, Yale University Press, New Haven, 1948, pp.55–349.

[8] A. Münster, Classical Thermodynamics, Wiley-Interscience, London-NewYork-Sydney-Toronto,1970.

[9] H. B. Callen, Thermodynamics and an Introduction to Thermostatics, seconded., Wiley, NewYork,1985.

[10] Rupperiner G, Phys. Rev. D,78 ,024016 (2008).

[11] Hickman, Y. Mishin, Atomistic modeling of grain boundary pre-melting in alloy systems, (2015).

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APPENDIX 5

SOME EQUIVALENT MODELS OF BLACK HOLES AND THEIR VALIDATION

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Abstract: The historical back ground of black hole region is presented here. The properties and characteristics of black holes are studied. The equivalent models of the black holes are proposed. Some models are found to act in similar ways as that of the black holes. The features of the models validate the existence of black holes identified within the galaxies. These models would be very useful in predicting different behaviors of the black holes found in the galaxies.

Key words: The black holes, singularity, event horizon, galaxies.

Historical background: -The scientists of present era working in the field of astronomical science have discovered some most powerful and delightful events like the birth of latest solar systems to the cataclysmic death of massive stars occurring within the nature. The black holes lead a crucial role in the formation and in the demise of the universe. Still, most of the properties of the black holes and their physics are subject of intensive study. Since there are numerous sorts of black holes discovered with various different characteristics. Some black holes are found to be of very big size, some are of small size, some are with electric charge, some are without electric charge, some black holes have rapid rotations, and a few are more sedentary. Therefore, the scientists haven't found any general and perfect theory which could explain the foremost of the characteristics found within the black holes. Once we attempt to study the black holes from an outsized distance it's surprisingly just a huge object. If we suppose a region having mass like the mass of the sun, it'll exert pulling force on us almost like that of the sun. All that's missing is that the heat and light wave and radiations that the sun applies. The region itself is singularity. It is some extent where density of fabric becomes infinitely large. We cannot see the singularity itself. it's shrouded by event horizon which is usually considered because the surface of the region. However, the event horizon isn't a true physical boundary. it's not a membrane or a surface. it's simply defined as a specific distance from the singularity point. If something happens to fall below this threshold distance, it cannot get out from this region. it's the space from the singularity where the gravitational pull is so extreme that nothing can go out the region clutches. it's been found that even light cannot shake the black holes. Black holes are alleged to be made from extremely massive substances with immense gravity that doesn't allow anything to flee. Even light cannot escape the black holes. Several theories are predicted for the formation of black holes. The classical theory of black hole region is given as the end state of a huge star. Most of the scientists realize it as a dead state of stars or the remains of dead stars. When the activity of emission of warmth, light and radiation from a huge star ceases due to lack of active materials, the residual state of it's referred to as theblack region. We will say that the body state of a huge star forms the region. The second prediction of formation of the black hole region is by collision process. When two very massive stars like neutrons stars collide and merge together, they form the black holes. Black holes that form through these mechanisms are reported to have masses usually three to 10 times greater than the mass of the Sun and that they are called stellar-mass black holes. But there have also been found super massive black holes which are greater than million to a billion times the mass of our Sun. They have been found at the center of just about all massive galaxies. What's the mechanism of their formation remains not fully explained. However, it's documented that some states of stars are responsible to create the black hole region. The lifetime of a star is often understood as endless struggle between gravity and pressure radiation. The gravity force acting within the substance tends to compress the star within it, but the radiation produced due to nuclear fusion continuously taking place within it pushes its matter outwards. A kind of virtual equilibrium is established between these two counter effects. But when the active fuel within the star is exhausted, the star stops burning. The balance condition of the gravity force and the out-ward force due to -radiation pressure is disturbed. In due course of time nothing can counteract gravity and therefore the whole mass is squeezed into some extent of zero volume and infinite density. This is called the singularity point. Now it is understood that the singularity point is surrounded by a boundary. This boundary is understood as an event horizon. The entire system is named as the black hole. The interior phenomena depend on the masses of the black holes. The Indian–American astrophysicist Subrahmanyan Chandrasekhar first Speculated in 1930 that the fate of massive stars might not be just to chill down, as smaller stars do, but they will also collapse into something denser objects like white dwarfs and neutron stars. In 1939, the American physicists Oppenheimer and Hartland Snyder predicted that black holes might be formed in nature from the collapse of massive stars. In 1974, British theoretical physicist hawking applied the scientific theory. He discovered that quantum black holes aren't as black as classical black holes. They emit thermal radiation. The term "black hole" was coined in 1967 by the American physicist John Arichbald Wheeler. Theoretically the black holes are very simple to explain. There are three parameters sufficient to characterize them fully: mass, momentum, which describes the rotation happening within the black holes. The third component of the black holes is taken as the electric charge. Considering the stars, from which the black holes are formed, these are only a few characteristics, leading physicists to mention that "black holes haven't any hair". Astrophysical black holes are very simple because they're alleged to haven't any charges. If they might have charge, then they might be quickly neutralized by the encompassing plasma. Schwarzschild's solution to Einstein's equations reveals that a region isn't rotating, but Kerr's solution shows that black whole are going to be rotating. it's reasonable to think that black holes rotate: if they have been formed from a rotating collapsing star or from the merger of two neutron stars, they need to have conserved their rotational energy. Moreover, they will spin up successive interactions. It had been once thought that black holes don't emit anything. However, Hawking discovered that if quantum effects are taken under consideration, it can be shown that black holes can radiate gravitational energy, thermal energy and particles also. Since, Hawking radiation carries energy far away from the region so this reduces its mass consistent with the formula ($E = mc^2$). Therefore, a region shrinks and eventually it's evaporated. This effect is vital for very tiny black holes, but astrophysical black holes are very massive and that they would need a time much longer than the age of the Universe to evaporate. consistent with Hawkins, the planet of black holes is filled with surprises: there could also be black holes smaller than a flea and lighter than a feather and tiny black holes could within the future at some point be produced in the laboratory. Primordial black holes may have existed during the first stages of the Universe, a couple of moments after the large Bang took place. At that point the temperature and pressure were so high that they might have squeezed the matter right down to a singularity point. However, only primordial black holes of a few billion tons could have survived until this day; the lighter ones would all have evaporated. The lightest black holes are referred to as elementary black holes. With the startup of the massive Hadron Collider (LHC) these microscopic black holes received much attention as they could be produced during particle collisions at the LHC. However, they might be disintegrated immediately again. They might not have enough time to grow into matter and cause any macroscopic effects. it's the quantum physics which will explain how black holes die. In quantum physics, subatomic positive particles and negative antiparticles explode into existence all the time. Since the positive particles have positive mass and therefore the negative antiparticles have an opposite negative mass, they will cancel one another effects, and this happens insignificant way. But if these particles and antiparticles came into existence next to a region there, it may happen that they are doing an equivalent cancellation. Famed English physicist Hawking theoretically showed that something happens differently around a region. The thought is that particles and antiparticles might not be ready to automatically cancel one another, due to the black hole's gravity. It pulls the negative antiparticle into black hole-oblivion, and leaves the positive particle alone and "uncancelled," making it "real." These positive particles then are emitted from the region. This phenomenon is named Hawking Radiation. But that's not the top of the method. After an extended time, the region would start losing mass thanks to the gradual addition of antiparticles. According to Hawking the black holes would evaporate in suitable conditions. During evaporation, the region emits energy within the sort of the positive particles that escape. The quantity of energy release will depend upon the quantity of mass of the region. The more energy would be released from massive region. The region would eventually lose such a lot mass that it might become small and unstable. The region would then lose the remainder of its mass during a short interval of time. This may happen as abrupt explosions. These explosions could also be detected as gamma radiation bursts. So, we will say that black holes perish in this manner. They are doing so when the theories of the acutely large close with the theories of the extreme small. They are doing so slowly, then all directly.

Propositions of black holes: - 1: explosion Theory: -If we assume that the black holes were produced at the time of massive bang, it was the giant black hole itself liable for this phenomenon. It burst itself to optimize its energy and to urge stability for the nonce. On applied mathematics it has been shown that the dimensions of component black holes ranges from micro miniatures to larger than million size of the sun. Further the method of fission reaction happened and really massive black holes continuously went on breaking in suitable circumstances. Thus, energy evolved to the galaxy. The black holes of micro size went on evaporating through hawking evaporation process, which further gives off energy within the sort of radiations and warmth the galaxy. This process is sustained till date.

2: - Transducer theory: - If we assume that region is found in nature as a tool which has ability to rework matter in to energy and energy in to matter. It’s a bit like an engine. It takes matter from surrounding converts some a part of it into energy and therefore the remains are shipped to the singularity point. during this process the equilibrium will be achieved almost like Carnot’s cycle.

3: - Dual nature theory: If we assume that the region is a tool which could take all the masses and energy radiations near its event horizon and sends it to the singularity point. The whole thing is situated at the center of the galaxy , then we'll need to assume another twin galaxy. If the primary nature device is named a black hole, then its twin counter a part of twin galaxy is going to be called the white galaxy. The white hole galaxy will serve in other way. Thus, a pair of holes will lead the pair of galaxies.

4: - The region is a pool for transportation of mass and energy between two galaxies: If we assume that there are existed always twin galaxies in nature, then the pair of regions one called black hole while other called white hole function a pool between the two galaxies. The mass and radiation of the first galaxy is transported to a different galaxy in this process. The thought and views given in the proposition are based on intensive study of article related to the black holes.

Acknowledgements - The idea of such models have been drawn from the literatures provided by the following scientists. We are very much grateful to them.

1:Stephen Hawking, who provided a theoretical argument for existence of black hole in 1974."Black hole explosions",

2: The Indian–American astrophysicist Subrahmanyan Chandrasekhar, literatures on white dwarfs and neutron stars.

3:Frank Helie, Ph.D. Physics, Stanford University,

References: -

1. [1] Don N Page**,** Department of Physics, Institute for Theoretical Physics, University of Alberta, Edmonton, AB T6G 2J1, Canada, E-mail: don@phys.ualberta.ca

   New Journal New Journal of Physics **7** (2005) 203Received 03 September 2004 Published 29 September 2005 Online at http://www.njp.org/doi:10.1088/1367-2630/7/1/203

2. More S S 2004 Higher order corrections to black hole entropy *Preprint* gr-qc/0410071
3. Amelino-Camelia G, Arzano M and Procaccini A 2004 *Phys. Rev.* D **70** 107501
4. Bousso R 2004 *J. High Energy Phys.* JHEP03(2004)054
5. Thorne K S, Zurek W H and Price R H 1986 *Black Holes: The Membrane Paradigm* ed K S Thorne, R H Price and D A MacDonald (New Haven, CT:Yale University Press) p 280
6. A. Tchekhovskoy, R. Narayan and J. C. McKinney, Mon. Not. Roy. Astron. Soc. 418, L79 (2011).
7. T. Igata, T. Harada and M. Kimura, Phys. Rev. D 85, 104028 (2012).
8. V. P. Frolov, Phys. Rev. D 85, 024020 (2012)

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